## Basic Statistics and Probability (2018-19) <br> Science \& Engineering Program Boston University-Faculty of Science UAM

## Chapter 4: Discrete Random Variables

The following problems are from McClave, J. and Sincich, T. (2017), Statistics, 13th. edition, Pearson. The number preceding the exercise is the corresponding one from this textbook. Some of the problems may also appear in previous editions of the book, possibly with different numbering.
4.7 Customers in line at a Subway shop. The number of customers, $x$, waiting in line to order sandwiches at a Subway shop at noon is of interest to the store manager. What values can $x$ assume? Is $x$ a discrete or continuous random variable?
4.9 Mongolian desert ants. Refer to the Journal of Biogeography (Dec. 2003) study of ants in Mongolia, presented in Exercise 2.68. Two of the several variables recorded at each of 11 study sites were annual rainfall (in millimeters) and number of ant species. Identify these variables as discrete or continuous.
4.19 A discrete random variable $x$ can assume five possible values: $20,21,22,23$, and 24 . The following MINITAB histogram shows the likelihood of each value.

a. What is $p(22)$ ?
b. What is the probability that $x$ equals 20 or 24 ?
c. What is $P(x \leq 23)$ ?
4.20 Explain why each of the following is or is not a valid probability distribution for a discrete random variable $x$ :
a.

| $x$ | 0 | 1 | 2 | 3 |
| :--- | ---: | ---: | ---: | ---: |
| $p(x)$ | .2 | .3 | .3 | .2 |
| $x$ | -2 | -1 | 0 |  |
| $p(x)$ | .25 | .50 | .20 |  |
| $x$ | 4 | 9 | 20 |  |
| $p(x)$ | -.3 | 1.0 | .3 |  |
| $x$ | 2 | 3 | 5 | 6 |
| $p(x)$ | .15 | .20 | .40 | .35 |

4.23 Toss three fair coins, and let $x$ equal the number of heads observed.
a. Identify the sample points associated with this experiment, and assign a value of $x$ to each sample point.
b. Calculate $p(x)$ for each value of $x$.
c. Construct a probability histogram for $p(x)$.
d. What is $P(x=2$ or $x=3)$ ?
4.27 Downloading apps to your cell phone. According to a study, nearly $40 \%$ of adult cell phone owners have downloaded an application ("app") to their cell phone. The accompanying table gives the probability distribution for $x$, the number of apps used at least once a week by cell phone owners who have downloaded an app to their phone.

| Number of Apps Used, $x$ | $p(x)$ |
| :--- | ---: |
| 0 | .30 |
| 1 | .24 |
| 2 | .06 |
| 3 | .06 |
| 4 | .05 |
| 5 | .10 |
| 6 | .07 |
| 7 | .05 |
| 8 | .03 |
| 9 | .02 |
| 10 | .02 |

a. Show that the properties of a probability distribution for a discrete random variable are satisfied.
b. Find $P(x \geq 10)$ and $P(x>3)$.
4.46 Consider the probability distribution shown here:

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $p(x)$ | .02 | .07 | .10 | .15 | .30 | .18 | .10 | .06 | .02 |

a. Calculate $\mu, \sigma^{2}$ and $\sigma$.
b. Graph $p(x)$. Locate $\mu, \mu-2 \sigma$, and $\mu+2 \sigma$ on the graph.
c. What is the probability that $x$ will fall into the interval $x \pm 2 \sigma$ ?
4.48 Downloading apps to your cell phone. Refer to the survey of cell phone owners, Exercise 4.27. There you were given the probability distribution for $x$, the number of apps used at least once a week by cell phone owners who have downloaded an app to their phone.
a. Find the mean and variance of $x$.
b. Give an interval that will contain the value of $x$ with a probability of at least .75 .
4.63 If $x$ is a binomial random variable, compute $p(x)$ for each of the following cases:
a. $n=5, x=1, p=.2$
b. $n=4, x=2, q=.4$
c. $n=3, x=0, p=.7$
d. $n=5, x=3, p=.1$
e. $n=4, x=2, q=.6$
f. $n=3, x=1, p=.9$
4.64 If $x$ is a binomial random variable, find the following probabilities:
a. $P(x=2)$ for $n=10, p=.4$
b. $P(x \leq 5)$ for $n=15, p=.6$
c. $P(x>1)$ for $n=5, p=.1$
4.72 Analysis of bottled water. Is the bottled water you're drinking really purified water? A study of various brands of bottled water conducted by the Natural Resources Defense Council found that $25 \%$ of bottled water is just tap water packaged in a bottle (https://www.nrdc.org/). Consider a sample of five bottled-water brands, and let $x$ equal the number of these brands that use tap water.
a. Explain why $x$ is (approximately) a binomial random variable.
b. Give the probability distribution for $x$ as a formula.
c. Find $P(x=2)$.
d. Find $P(x \leq 1)$.

