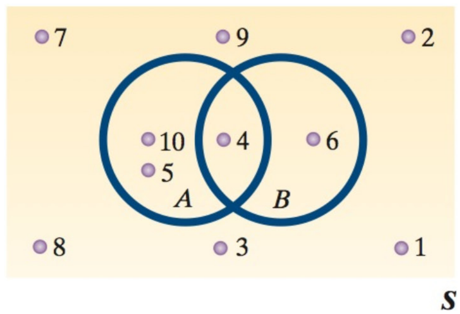
Basic Statistics and Probability (2018-19)
Science & Engineering Program Boston University-Faculty of Science UAM

Chapter 3: Probability

The following problems are from McClave, J. and Sincich, T. (2017), *Statistics*, 13th. edition, Pearson. The number preceding the exercise is the corresponding one from this textbook. Some of the problems may also appear in previous editions of the book, possibly with different numbering.

3.10 The following Venn diagram describes the sample space of a particular experiment and events A and B:



a. Suppose the sample points are equally likely. Find $P(A)$ and $P(B)$.
b. Suppose $P(1) = P(2) = P(3) = P(4) = P(5) = 1/20$ and $P(6) = P(7) = P(8) = P(9) = P(10) = 3/20$. Find $P(A)$ and $P(B)$.

3.14 Two fair dice are tossed, and the up face on each die is recorded.

a. List the 36 sample points contained in the sample space.
b. Assign probabilities to the sample points.
c. Find the probability of observing each of the following events:

A: { A 3 appears on each of the two dice. }
B: { The sum of the numbers is even. }
C: { The sum of the numbers is equal to 7. }
D: { A 5 appears at least on one of the dice. }
E: { The sum of the numbers is 10 or more. }

3.15 Two marbles are drawn at random and without replacement from a box containing two blue marbles and three red marbles.

a. List the sample points.
b. Assign probabilities to the sample points.
c. Determine the probability of observing each of the following events:

A: { Two blue marbles are drawn. }
B: { A red and a blue marble are drawn. }
C: { Two blue marbles are drawn. }

3.22 African rhinos. Two species of rhinoceros native to Africa are black rhinos and white rhinos. The International Rhino Federation estimates that the African rhinoceros population consists of 5,055 white rhinos and 20,405 black rhinos. Suppose one rhino is selected at random from the African rhino population and its species (black or white) is observed.

a. List the sample points for this experiment.
b. Assign probabilities to the sample points on the basis of the estimates made by the International Rhino Federation.

3.31 Jai alai Quinella bet. The Quinella bet at the parimutuel game of jai alai consists of picking the jai alai players that will place first and second in a game, *irrespective of order*. In jai alai, eight players (numbered 1, 2, 3, ..., 8) compete in every game.

a. How many different Quinella bets are possible?
b. Suppose you bet the Quinella combination 2–7. If the players are of equal ability, what is the probability that you win the bet?

3.45 A fair coin is tossed three times, and the events A and B are defined as follows:

A: { At least one head is observed. }
B: { The number of heads observed is odd. }

a. Identify the sample points in the events A, B, $A \cup B$, A^c and $A \cap B$.
b. Find $P(A)$, $P(B)$, $P(A \cup B)$, $P(A^c)$ and $P(A \cap B)$ by summing the probabilities of the appropriate sample points.
c. Use the additive rule to find $P(A \cup B)$. Compare your answer with that for the same event in part **b.**
d. Are A and B mutually exclusive? Why?

3.51 The outcomes of two variables are (Low, Medium, High) and (On, Off), respectively. An experiment is conducted in which the outcomes of each of the two variables are observed. The probabilities associated with each of the six possible outcome pairs are given in the following table:

	Low	Medium	High
On	.50	.10	.05
Off	.25	.07	.03

Consider the following events:

A: { On }
B: { Medium or On }
C: { Off and Low }
D: { High }

a. Find $P(A)$.
b. Find $P(B)$.
c. Find $P(C)$.
d. Find $P(D)$.
e. Find $P(A^c)$.
f. Find $P(A \cup B)$.
g. Find $P(A \cap B)$.
h. Consider each possible pair of events taken from the events A, B, C, and D. List the pairs of events that are mutually exclusive. Justify your choices.

3.55 Gene expression profiling. Gene expression profiling is a state-of-the-art method for determining the biology of cells. In *Briefings in Functional Genomics and Proteomics* (Dec. 2006), biologists reviewed several gene expression profiling methods. The biologists applied two of the methods (A and B) to data collected on proteins in human mammary cells. The probability that the protein is cross-referenced (i.e., identified) by method A is .41, the probability that the protein is cross-referenced by method B is .42, and the probability that the protein is cross-referenced by both methods is .40.

a. Draw a Venn diagram to illustrate the results of the gene-profiling analysis.
b. Find the probability that the protein is cross-referenced by either method A or method B.
c. On the basis of your answer to part **b.**, find the probability that the protein is not cross-referenced by either method.

3.61 Fighting probability of fallow deer bucks. In *Aggressive Behavior* (Jan./Feb. 2007), zoologists investigated the likelihood of fallow deer bucks fighting during the mating season. During the observation period, the researchers recorded 205 encounters between two bucks. Of these, 167 involved one buck clearly initiating the encounter with the other. In these 167 initiated encounters, the zoologists kept track of whether or not a physical contact fight occurred and whether the initiator ultimately won or lost the encounter. (The buck that is driven away by the other is considered the loser.) A summary of the 167 initiated encounters is provided in the table

	Initiator Wins	No Clear Winner	Initiator Loses	Totals
Fight	26	23	15	64
No Fight	80	12	11	103
Totals	106	35	26	167

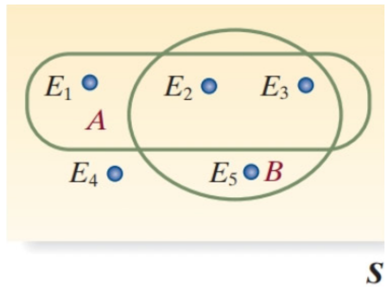
Suppose we select one of these 167 encounters and note the outcome (fight status and winner).

a. What is the probability that a fight occurs and the initiator wins?
b. What is the probability that no fight occurs?
c. What is the probability that there is no clear winner?
d. What is the probability that a fight occurs or the initiator loses?
e. Are the events "No clear winner" and "Initiator loses" mutually exclusive?

3.73 For two events, A and B, $P(A) = .4$, $P(B) = .2$, and $P(A \cap B) = 0.1$.

a. Find $P(A|B)$.
b. Find $P(B|A)$.
c. Are A and B independent events?

3.77 Consider the experiment defined by the accompanying Venn diagram, with the sample space S containing five sample points. The sample points are assigned the following probabilities: $P(E_1) = .1$, $P(E_2) = .1$, $P(E_3) = .2$, $P(E_4) = .5$, $P(E_5) = .1$.



a. Calculate $P(A)$, $P(B)$ and $P(A \cap B)$.
b. Suppose we know that event A has occurred, so the reduced sample space consists of the three sample points in A: E_1, E_2, and E_3. Use the formula for conditional probability to determine the probabilities of these three sample points given that A has occurred. Verify that the conditional probabilities are in the same ratio to one another as the original sample point probabilities and that they sum to 1.
c. Calculate the conditional probability $P(B|A)$ in two ways: First, sum $P(E_2|A)$ and $P(E_3|A)$, since these sample points represent the event that B occurs given that A has occurred. Second, use the formula for conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Verify that the two methods yield the same result.

3.93 Are you really being served red snapper? Red snapper is a rare and expensive reef fish served at upscale restaurants. Federal law prohibits restaurants from serving a cheaper, look-alike variety of fish (e.g., vermillion snapper or lane snapper) to customers who order red snapper. Researchers at the University of North Carolina used DNA analysis to examine fish specimens labeled "red snapper" that were purchased from vendors across the country (*Nature*, July 15, 2004). The DNA tests revealed that 77% of the specimens were not red snapper, but the cheaper, look-alike variety of fish.

a. Assuming that the results of the DNA analysis are valid, what is the probability that you are actually served red snapper the next time you order it at a restaurant?
b. If there are five customers at a restaurant, all who have ordered red snapper, what is the probability that at least one customer is actually served red snapper?

3.94 Fighting probability of fallow deer bucks. Refer to the *Aggressive Behavior* (Jan./Feb. 2007) study of fallow deer bucks fighting during the mating season, presented in Exercise 3.61. Recall that researchers recorded 167 encounters between two bucks, one of which clearly initiated the encounter with the other. A summary of the fight status of the initiated encounters was provided in the accompanying table of Exercise 3.61. Suppose we select 1 of these 167 encounters and note the outcome (fight status and winner).

a. Given that a fight occurs, what is the probability that the initiator wins?
b. Given no fight, what is the probability that the initiator wins?
c. Are the events "no fight" and "initiator wins" independent?

3.115 Picking a basketball team. Suppose you are to choose a basketball team (five players) from eight available athletes.

a. How many ways can you choose a team (ignoring positions)?
b. How many ways can you choose a team composed of two guards, two forwards, and a center?
c. How many ways can you choose a team composed of one each of a point guard, shooting guard, power forward, small forward, and center?

3.136 Suppose the events B_1, B_2, and B_3 are mutually exclusive and complementary events such that $P(B_1) = .2$, $P(B_2) = .15$, and $P(B_3) = .65$. Consider another event A such that $P(A|B_1) = .4$, $P(A|B_2) = .25$, and $P(A|B_3) = .6$. Use Bayes's rule to find

a. $P(B_1|A)$
b. $P(B_2|A)$
c. $P(B_3|A)$

3.139 Drug testing in athletes. When Olympic athletes are tested for illegal drug use (i.e., doping), the results of a single test are used to ban the athlete from competition. In *Chance* (Spring 2004), University of Texas biostatisticians demonstrated the application of Bayes's rule to making inferences about testosterone abuse among Olympic athletes. They used the following example: In a population of 1,000 athletes, suppose 100 are illegally using testosterone. Of the users, suppose 50 would test positive for testosterone. Of the nonusers, suppose 9 would test positive.

a. Given that the athlete is a user, find the probability that a drug test for testosterone will yield a positive result. (This probability represents the sensitivity of the drug test.)
b. Given that the athlete is a nonuser, find the probability that a drug test for testosterone will yield a negative result. (This probability represents the specificity of the drug test.)
c. If an athlete tests positive for testosterone, use Bayes's rule to find the probability that the athlete is really doping. (This probability represents the *positive predictive value* of the drug test.)