Basic Statistics and Probability (2018-19)
Science & Engineering Program Boston University-Faculty of Science UAM

QUIZ 2. SOLUTIONS

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1. Is there enough sample evidence supporting the claim of the manufacturer that the true average fuel efficiency μ is at least 30 mpg?

a) In the alternative hypothesis we place the statement for which we want to have sample evidence before we accept it:

$$H_0: \quad \mu \le 30$$
$$H_1: \quad \mu > 30$$

As the sample size is $n = 6$ (small; we cannot apply CLT), we have to assume that X = "fuel efficiency of a car from that manufacturer" follows a distribution N(μ,σ).

b) $\bar{x} = 30.05$ is a point estimate of μ and $s = 0.99$ is a point estimate of σ. The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{30.05 - 30}{0.99/\sqrt{6}} = 0.124.$$

c) The rejection region of the test given in (a) for $\alpha = 0.01$ is $R = \{t > t_{5;0.01}\} = \{t > 3.365\}$.

d) Since $t = 0.124 < 3.365$, there is not enough sample evidence to reject $H_0 : \mu \le 30$ at the 1% significance level. The sample does not support the claim of the manufacturer.

e) For any $\alpha < 0.4531$ (including the usual values 0.01, 0.05 and 0.1), there is not enough evidence to reject H_0. Consequently, the p-value is considered to be "large" and it is not reasonable to reject H_0.

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2. a) By the Central Limit Theorem, the approximate sampling distribution of the sample mean $\bar{X} = \sum_{i=1}^{100} X_i/100$ is normal. The expectation is $E(\bar{X}) = E(X) = \mu = 200$ and the standard deviation is s.d.$(\bar{X}) = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{100}} = 2$.

b) By the result in (a), the approximate probability that the sample mean cholesterol level $\bar{X}$ is lower than 201 is

$$P\{\bar{X} < 201\} \simeq P\left\{Z < \frac{201 - 200}{2}\right\} = P\{Z < 0.5\} = 0.5 + 0.1915 = 0.6915.$$

where $Z \sim N(0, 1)$.

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3. a) $\hat{p} = 12/35 = 0.3429$

b) A 95% confidence interval for p:

$$\mathrm{CI}_{95\%}(p) = \left(\hat{p} \mp z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = \left(0.3429 \mp 1.96\sqrt{\frac{0.3429(1-0.3429)}{35}}\right) = (0.3429 \mp 0.1573)$$

c) We are asked to find n such that the half-length of the $\mathrm{CI}_{95\%}(p)$ is lower than .05.
First possible solution:

$$1.96\sqrt{\frac{0.3429(1-0.3429)}{35}} < 0.05 \Rightarrow n > 346.2$$

Second possible solution:

$$1.96\sqrt{\frac{0.5^2}{35}} < 0.05 \Rightarrow n > 384.16$$