

Name: \_\_\_\_\_

**Basic Statistics and Probability (2018-19)**  
**Science & Engineering Program Boston University-Faculty of Science UAM**

**MID-TERM**

The total maximum score in this quiz is 10 points. The weight of this quiz in the final grade is 30%.

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**1. (Total max. score = 2 points)** A pharmacist has filled a box with six different kinds of antibiotic capsules. There are a total of 300 capsules, which are distributed as follows: tetracycline (15), penicillin (30), minocycline (45), Bactrim (60), streptomycin (70), and Zithromax (80). She asks her assistant to mix the pills thoroughly and to withdraw a single capsule from the box.

**a) (0.5 points)** List the sample points (simple events) for the experiment of withdrawing a single capsule from the box. Compute the probability of each sample point.

**b) (0.25 points)** What is the probability that the capsule selected is either penicillin or streptomycin?

**c) (0.25 points)** What is the probability that the capsule selected is neither Zithromax nor tetracycline?

**d) (0.25 points)** What is the probability that the capsule selected is not penicillin?

**e) (0.75 points)** Now the pharmacist asks her assistant to draw **two** capsules from the box **without** replacement and to check whether they are Zithromax or not. Determine the probability that two Zithromax capsules are drawn. Determine the probability that one of capsules is Zithromax and the other one is not.

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**2. (Total max. score = 1.25 points)** A certain casino uses two kinds of dice. One kind of die is fair and is used 99% of the time. The unfair die rolls a six 50% of the time and is used for the rest of the time. The other faces (1 to 5) of the unfair die are equally probable among themselves.

**a) (0.25 points)** What is the probability that the die will roll a two if we choose the unfair die?

**b) (0.5 points)** Use the Total Probability Rule to answer the following question: if we pick up a single die at random, how likely is it that we will roll a six?

**c) (0.5 points)** Use Bayes Rule to answer the following question: if we roll one of the two dice at random and obtain a six, what is the probability that the rolled die is the unfair one?

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**3. (Total max. score = 0.75 points)** Suppose  $X$ , the lifetime (in years) of an electrical device, is a random variable following a uniform probability distribution on the interval  $[c, d] = [1, 10]$ .

**a) (0.25 points)** Draw the probability density of  $X$ .

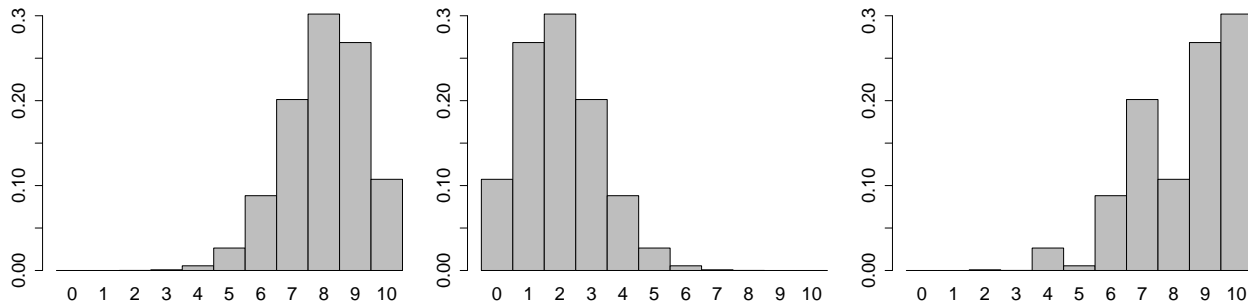
**b) (0.25 points)** What is the probability that the device lasts less than 4 years?

**c) (0.25 points)** What is the probability that the lifetime of the device is between 4 and 7 years?

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**4. (Total max. score = 2.75 points)** In a Pew Research Center survey conducted March 27-April 9, 2018, eight-in-ten (80%) Americans said the International Space Station had been a good investment for the country, while two-in-ten said it had been not. Consider a group of 10 Americans who are asked whether the space station has been a good investment. Let  $X$  be the number of them who think it has.

- a) (0.25 points) What is the probability distribution of  $X$ ? Is  $X$  discrete or continuous?
- b) (0.25 points) Give the probability distribution of  $X$  as a formula.
- c) (0.5 points) Compute  $P\{X = 7\}$ .
- d) (0.25 points) What is the probability that all the ten Americans surveyed think the space station has been a good investment?
- e) (0.25 points) What is the probability that  $X$  equals 7 or 10?
- f) (0.5 points) Find  $P\{X \leq 8\}$ .
- g) (0.5 points) Find the population mean and variance for  $X$ .
- h) (0.25 points) One of the following three histograms is the one corresponding to the probability mass function of  $X$ . Choose the correct one and justify your choice.



**5. (3.25 points)** After a full-term pregnancy, the length  $X$  (in cm) of a healthy newly born female baby in Spain follows a normal distribution with mean  $\mu = 50$  and standard deviation  $\sigma = 1.5$ . We denote by  $Z$  a random variable with a standard normal distribution.

- a) (0.25 points) Find  $P\{0 < Z < 0.67\}$ .
- b) (0.25 points) Find  $P\{Z > 0.67\}$ .
- c) (0.5 points) Find  $P\{-1.33 < Z < 0.67\}$ .
- d) (0.5 points) Find a value  $c$  such that  $P\{Z > c\} = 0.15$ .
- e) (0.25 points) Find a value  $c$  such that  $P\{-c < Z < c\} = 0.7$ .
- f) (0.5 points) Find the probability that the length of a newly born girl is less than 52 cm.
- g) (0.5 points) Find the probability that the length of a newly born girl is between 49 and 52 cm.
- h) (0.5 points) Find a value  $c$  such that  $P\{X > c\} = 0.15$ .