$\qquad$

# Basic Statistics and Probability (2018-19) <br> Science \& Engineering Program Boston University-Faculty of Science UAM 

## FINAL EXAM

The total maximum score in this quiz is 10 points. The weight of this quiz in the final grade is $40 \%$.

1. (Total max. score $=2.5$ points) To study the relationship between intracellular calcium and blood pressure, researchers measured the ionized calcium concentration in blood platelets of 39 normotensive people and 44 hypertensive people. The results are given in Table 1.

|  |  | Calcium concentration |  |
| :--- | :---: | :---: | :---: |
| Blood Pressure | $n$ | Mean | Std deviation |
| Normal | 39 | 108.1 | 17.1 |
| High | 44 | 168.3 | 25.6 |

Table 1
a) ( 0.5 points) Find a $90 \%$ confidence interval for the mean ionized calcium concentration in people with normal blood pressure.
b) (1 point) Construct a $95 \%$ confidence interval for the difference between the mean ionized calcium concentration in people with normal and with high blood pressure.
c) (1 point) Is there enough sample evidence that the mean ionized calcium concentration is different in the two groups? Use $\alpha=.05$.
2. (Total max. score $=\mathbf{2 . 7 5}$ points) A company claims that they have a new treatment for a disease in trees and that $70 \%$ of diseased trees that receive the treatment improve their condition. In order to check this claim, a researcher applied the treatment to 84 diseased trees and saw that 66 of the trees had improved condition after two weeks.
a) ( 0.5 points) Describe the Bernoulli experiment carried out in this problem. Identify the parameter of interest. Compute a point estimate of this parameter.
b) ( $\mathbf{1}$ point) At the $1 \%$ significance level, test if there is enough sample evidence that the proportion of trees that improve their condition after the treatment is larger than $70 \%$.
c) ( $\mathbf{0 . 2 5}$ points) For the test in $\bar{b}$, is the p-value greater or lower than 0.01 ? Justify your answer (just saying "greater" or "lower" will not be deemed correct).
d) ( $\mathbf{1}$ point) Imagine that the true proportion of diseased trees improving their condition after the treatment is $60 \%$. We apply the treatment to 5 diseased trees. What is the probability distribution of the number of trees of those 5 that improve their condition after being treated? What is the probability that 3 or less of the 5 trees will improve? What is the probability that 3 or more of the 5 trees will improve?
3. (Total max. score $=\mathbf{1 . 7 5}$ points) In each subject participating in a certain study, resting heart rate was measured. The subject then drank 6 ounces of coffee. Ten minutes later their heart rates were measured again. The change in heart rate, $X$, followed a normal distribution with a mean increase of 7 beats per minute and a standard deviation of 10 .
a) ( 0.25 points) What is the probability distribution of the $z$-score of $X$ ?
b) ( 0.75 points) Find $P\{X>11\}$.
c) ( 0.75 points) Find a value $c$ such that $P\{7 \leq X \leq c\}=0.3413$.
4. (Total max. score $=\mathbf{1 . 2 5}$ points) Economists frequently predict profits under alternative economic scenarios. In a certain business the possible predictions for inflation are "high" or "low" and unemployment could be "high" or "low". There are four possible future scenarios (events), with the probabilities given in Table 2 .

| Scenario | Inflation | Unemployment | Probability |
| :---: | :---: | :---: | :---: |
| 1 | High | High | 0.16 |
| 2 | High | Low | 0.24 |
| 3 | Low | High | 0.36 |
| 4 | Low | Low | 0.24 |

Table 2
a) ( 0.25 points) What is the probability of high inflation? What is the probability of high unemployment?
b) ( 0.25 points) What is the probability of an scenario with simultaneously high inflation and high unemployment? Are high inflation and high unemployment independent events?
c) ( 0.25 points) What is the probability of high inflation conditional on unemployment being high?
d) ( 0.25 points) What is the probability of an scenario with high inflation or high unemployment?
e) ( $\mathbf{0 . 2 5}$ points) Are events "high inflation" and "high unemployment" mutually exclusive?
5. (Total max. score $=\mathbf{1 . 7 5}$ points) A sample of 10 boxes of raisins has these weights (in g ):

$$
25,28,29,29,30,34,35,35,37,38 .
$$

The boxplot of the weights is given in Figure 1 .

## Figure 1


a) (0.25 points) Identify the variable measured in the study and its type (quantitative or qualitative).
b) ( 0.25 points) Knowing that the sum of the 10 weights is 320 g , compute the value of the sample mean, $\bar{x}$.
c) ( 0.25 points) What is the median of the data set?
d) ( 0.25 points) What are the upper and lower quartiles of the data set?
e) ( 0.25 points) Is the sample symmetric or skewed? Why?
f) ( 0.5 points) If I substitute the value 38 by 100 , what happens to the sample mean? What happens to the median?

