Seminario de Análisis y Aplicaciones

Viernes, 29 de septiembre de 2017

11:30 h., Módulo 17 - Aula 520 (Depto. Matemáticas UAM)

Davide Barbieri

Universidad Autónoma de Madrid

Square-integrability for semidirect products

Resumen:

Some of the central objects for abstract harmonic analysis and generalized wavelet transforms are the matrix elements of unitary group representations and their properties as maps from Hilbert spaces to functions on the group. Given a unitary representation $\pi : G \to \mathcal{U}(\mathcal{H})$ of a locally compact group G on a separable Hilbert space $\mathcal{H}$, the continuous function on G given by $\mathcal{W}f(g) = \langle f, \pi(g)\psi \rangle_{\mathcal{H}}$ is the wavelet transform of $f \in \mathcal{H}$, with mother wavelet $\psi \in \mathcal{H}$. If π is irreducible, $\mathcal{W}$ is injective, and if, for a given ψ, $\mathcal{W}f$ belongs to $L^2(G)$ for all f, the representation is called *square integrable*. While having been deeply studied abstractly in representation theory, this property also implies that the wavelet transform is actually an isometry from $\mathcal{H}$ to $L^2(G)$, providing a canonical way to define an inversion formula as an integral on the group.

In many settings that are relevant for applications, however, irreducible representations are not square integrable. This happens in the Heisenberg group, the groups of Euclidean motions, and other semidirect products $G = N \rtimes H$, where N is a locally compact abelian group and H is a locally compact group that acts regularly on N by automorphisms. Most of the solutions proposed to recover square integrability consist of restricting to work on noninvariant subsets of the group, often with significant constraints on the mother wavelet.

We show that for any irreducible representation of such a group, which selects an orbit $\mathcal{O}$ of H in the Fourier dual (characters) group of N, there exists a nontrivial invariant subbundle of $\mathcal{O} \times H$, whose fibers are homeomorphic to the base, which defines a canonical measure space where a square integrability intrinsically holds. This recovers the original notion as a special case, and provides a reproducing formula as a restriction of the group Fourier transform.

Work in collaboration with Filippo De Mari and Ernesto De Vito.