SEMINARIO DE ANÁLISIS Y APLICACIONES

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Differentiability of Lipschitz functions mapping into quasi-Banach spaces.

Resumen:

Lipschitz maps between Banach spaces are smooth in many cases, which makes differentiability an invaluable tool in their Lipschitz classification. There are many important reasons for studying differentiability properties of Lipschitz functions between non-locally convex guasi-Banach spaces, and the possibility to linearize them with an eye to the study of their non-linear structure is one of them. When one wants to determine whether a given mapping between quasi-Banach spaces is differentiable at a point, the first thing to do is to investigate its directional differentiability, and this leads naturally to wonder whether Lipschitz functions from the real line into a quasi-Banach space are Frechet differentiable. Enflo felt wary about the usage of the Frechet derivative in the context of non-locally convex guasi-Banach spaces because it had one serious defect: the lacking of a mean value-type theorem. Indeed, local convexity is not only a sufficient condition for the mean value theorem to hold, but it is also necessary. The lack of local convexity in a quasi-Banach space X is also the responsible of other pathologies like the existence of continuously differentiable functions from (0,1) into X that fail to be Lipschitz. In this talk we analyze these shortcomings and connect them with the failure of the fundamental theorem of calculus in the setting of quasi-Banach spaces.

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