Topics in Effective Harmonic Analysis

Nikolai K. Nikolski (nikolski@math.u-bordeaux1.fr)

**Description:** This is an introduction to approximation problems and the invisible spectrum phenomenon arising in harmonic analysis. Special attention is paid to translations and dilations completeness related to the Riemann hypothesis. The invisible spectrum problem is as follows: a transformation, or an element of an algebra, is given by its "visible action" on basic elements (eigenvalues for matrices/operators, Fourier-Gelfand transforms for functions), and one is looking for a reconstruction of the entire spectrum and for estimates for inverses (resolvents) in terms of the spectral parameters.

The following themes might be included.

1. Finite Muckenhoupt bases and the Kreiss Matrix Theorem.
2. Invisible spectrum of Fourier multipliers (including multipliers of $A_2$ Muckenhoupt bases).
3. Condition numbers of large matrices and analytic capacities.
4. Corona theorems for $H^\infty$ trace algebras.
5. Invisible spectrum in the classical Wiener algebra (...why there exists no constructive proof for Wiener’s $1/f$ theorem?).

**REFERENCES/PREREQUISITE and GRADE CONDITIONS:**

The course is intended for graduate students. Acquaintance with the standard analysis concepts is welcome (on the level of Rudin’s textbooks listed below).

There is no required written work or regular readings. To receive a grade of Satisfactory in this course, students must attend regularly and participate in course discussions.