On the non-existence of some special eigenfunctions for the Dirichlet Laplacian and the Lamé system

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Abstract. Let Ω be a bounded Lipschitz domain in $\mathbb{R}^n$ with $n \geq 3$. We prove that the Dirichlet Laplacian does not admit any eigenfunction of the form $u(x) = \varphi(x') + \psi(x_n)$ with $x' = (x_1, \cdots, x_{n-1})$. The result is sharp since there are 2-d polygonal domains in which this kind of eigenfunctions does exist. These special eigenfunctions for the Dirichlet Laplacian are related to the existence of uniaxial eigenvibrations for the Lamé system with Dirichlet boundary conditions. Thus, as a corollary of this result, we deduce that there is no bounded Lipschitz domain in $3-d$ for which the Lamé system with Dirichlet boundary conditions admits uniaxial eigenvibrations.

Keywords: Dirichlet Laplacian, Eigenfunctions, Lamé system

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1. Introduction and main result

Let Ω be a bounded Lipschitz domain in $\mathbb{R}^n$ with $n \geq 2$.

We consider the Dirichlet eigenvalue problem

$$\begin{cases} -\Delta u = \gamma u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

We are interested in the existence of eigenfunctions u of (1.1) of the particular form

$$u(x) = \varphi(x') - \psi(x_n) \tag{1.2}$$

with $x' = (x_1, \cdots, x_{n-1}) \in \mathbb{R}^{n-1}$.

In 2 dimensions, i.e. when $n = 2$, it is easy to see that this kind of eigenfunctions does exist for some particular domains. It is sufficient to consider

$$u(x_1, x_2) = \cos(2\pi k x_1) - \cos(2\pi k x_2) \tag{1.3}$$

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for any $k \in \mathbb{N}$ in the square domain

$$\Omega = \left\{ (x_1, x_2) \in \mathbb{R}^2 : \mid x_1 \mid -1 < x_2 < 1 - \mid x_1 \mid \right\}. \tag{1.4}$$

Our main result asserts that this can not be the case in dimensions $n \geq 3$:

THEOREM 1.1. *Assume that $n \geq 3$. If Ω is a bounded Lipschitz domain of $\mathbb{R}^n$, then the Dirichlet eigenvalue problem* (1.1) *does not admit any non-trivial eigenfunction of the form* (1.2).

As indicated above, the result is false when $n = 2$ and (1.3)-(1.4) provides a counterexample.

The first step of the proof of Theorem 1.1 is to reduce the n-dimensional problem to the following $(n-1)$-dimensional one. Given any bounded domain $\omega \subset \mathbb{R}^{n-1}, n \geq 3$, show that there is no non-constant solution of

$$\begin{cases} -\Delta' \varphi = \gamma \varphi & \text{in } \omega, \\ \varphi = 1 & \text{on } \partial\omega, \\ \varphi \in C(\bar{\omega}), & \\ -1 \leq \varphi \leq 1. & \end{cases} \tag{1.5}$$

In (1.5) Δ' denotes the $(n-1)-$dimensional Laplacian in the variables $x' = (x_1, \cdots, x_{n-1})$.

When a non-constant solution $\varphi = \varphi(x')$ of (1.5) exists one sees that the $n-$dimensional domain

$$\Omega = \left\{ (x', x_n) : x' \in \omega, -\arccos(\varphi(x')) \leq \sqrt{\gamma} x_n \leq \arccos(\varphi(x')) \right\} \tag{1.6}$$

is such that the function

$$u = \varphi(x') - \cos\left(\sqrt{\gamma} x_n\right) \tag{1.7}$$

solves (1.1), u being of the form (1.2).

When $n = 2$, the counterexample (1.3)-(1.4) can in fact be built since non-constant solutions of the corresponding 1-d problem (1.5) do exist. It is easy to see that $\varphi(x_1) = \cos(2\pi k x_1)$ satisfies (1.5) in $\omega = (-1, 1)$ for $\gamma = 4\pi^2 k^2$.

Most of this paper is devoted to the proof of the fact that when $n \geq 3$, the only solution of (1.5) is the constant one:

THEOREM 1.2. *Assume that $n \geq 3$ and that ω is a bounded domain of $\mathbb{R}^{n-1}$. Then, the only solution of* (1.5) *with $\gamma \geq 0$ is $\gamma = 0$ and $\varphi = 1$.*

2. Application to the Lamé system

Before getting into the proof of these results let us point out that the problem under consideration is related to the existence of particular eigenfunctions of the Lamé system:

$$\begin{cases} -\mu\Delta w - (\lambda+\mu)\nabla \operatorname{div} w = \gamma w & \text{in} \quad \Omega, \\ w = 0 & \text{on} \quad \partial\Omega. \end{cases} \tag{2.1}$$

More precisely, let us analyze the existence of uniaxial eigenvibrations for system (2.1). Assume, for instance, that the eigenfunction $w = (w_1, \cdots, w_{n-1}, w_n)$ of (2.1) is of the form

$$w = (0, w_n), \tag{2.2}$$

i.e. $w_1 \equiv \cdots \equiv w_{n-1} \equiv 0$. Then w solves (2.2) if and only if $u = w_n$ solves

$$\begin{cases} -\mu\Delta' u - (\lambda+2\mu)\partial_n^2 u = \gamma u & \text{in} \quad \Omega, \\ u = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{2.3}$$

being of the form (1.2).

Indeed, the first $(n-1)$-equations of (2.1) reduce to $\partial_k\partial_n w_n = 0$ with $k = 1, \ldots, n-1$ yielding $\partial_n w_n = c(x_n)$. Hence $w_n(x) = \varphi(x') - \psi(x_n)$ for some functions φ and ψ.

By means of a change of variables, system (2.3) may be reduced to (1.1). According to the counterexample (1.3)-(1.4), when $n = 2$ non trivial eigenfunctions of (2.2) of the form (2.3) may exist in suitable polygonal domains. However, due to Theorem 1.1, this cannot be the case when $n \geq 3$. More precisely, the following holds:

THEOREM 2.1. *Let Ω be a bounded Lipschitz domain of $\mathbb{R}^n$ with $n \geq 3$. Then the unique solution w of the Lamé eigenvalue problem (2.1) of the form*

$$w = (0, \cdots, 0, w_n) \tag{2.4}$$

is the trivial one.

In other words, if w solves (2.1) being of the form (2.4), then necessarily $w \equiv 0$ in dimensions $n \geq 3$.

According to [5], the analysis of the existence of solutions of the form (2.2) for the time-dependent Lamé system for $t \in (0, T)$, $T > 0$ being large enough, may be reduced to the study of the eigenvalue problem (2.1)-(2.2). Therefore, Theorem 1.1 provides a negative answer to the existence of such solutions for the evolution problem when $n \geq 3$ (we refer to [5] for a discussion of the evolution problem). In [5], section

7.2, the existence of 3-dimensional domains with non-trivial solutions for (2.1)-(2.2) was claimed. However, the eigenfunctions in [5] do not satisfy the Dirichlet boundary conditions. Thus the example in [5] is incomplete and actually, as shown in Theorem 1.1 above, such domains do not exist.

The question whether or not uniaxial eigenvibrations exist for the Lamé system arises naturally when analyzing the asymptotic behavior of solutions of the system of magnetoelasticity in bounded domains (see [4]).

Explicit bounds on the first eigenvalue of the Lamé-system have been obtained by Kawohl and coauthor in [2]. When doing that for $n = 2$ one finds an eigenfunction as in (2.2) in Lemma 5 of [2].

The rest of the paper is devoted to the proof of Theorem 1.1. In section 3 we reduce the problem to the existence of solutions of (1.5). In section 4 we prove Theorem 1.2.

3. Reduction to the $(n-1)$-dimensional problem

In this section we reduce the proof of Theorem 1.1 to that of Theorem 1.2.

First of all we observe that, since u solves (1.1) and Ω is Lipschitz, then it follows that $u \in C(\bar{\Omega})$. According to this and in view of the decomposition (1.2), we deduce that both φ and ψ are continuous in the orthogonal projection of $\bar{\Omega}$ to $x' \in \mathbb{R}^{n-1}$ and $x_n \in \mathbb{R}$ that we shall denote by $\pi_{n-1}(\Omega)$ and $\pi_1(\Omega)$ respectively.

Combining (1.1)-(1.2) we deduce that

$$-\Delta'\varphi - \gamma\varphi = -\partial_n^2\psi - \gamma\psi = c \text{ in } \Omega$$

where $c \in \mathbb{R}$ is a constant. Thus, adding the constant c/γ to both φ and ψ we may assume that $c = 0$. Thus

$$-\Delta'\varphi = \gamma\varphi \text{ in } \pi_{n-1}(\Omega) \tag{3.1}$$

while

$$\psi(x_n) = \alpha \cos\left(\sqrt{\gamma}x_n + \beta\right)$$

for suitable constants $\alpha, \beta \in \mathbb{R}$. Without loss of generality we may rescale both the function as well as the domain and with an additional translation we may assume $\alpha = \gamma = 1$, $\beta = 0$ and hence we obtain

$$u(x) = \varphi\left(x'\right) - \cos\left(x_n\right). \tag{3.2}$$

In view of (3.2) the fact that Ω is bounded implies $\|\varphi\|_\infty \le 1$. Indeed, if $\varphi(x') > 1$ for some $(x', x_n) \in \Omega$, then $u > 0$ on $\bar{\Omega} \cap \{x'\} \times \mathbb{R}$ and

$u \in C_0(\bar{\Omega})$ implies $\{x'\} \times \mathbb{R} \subset \Omega$, which shows that Ω is unbounded. A similar argument applies if $\varphi(x') < -1$ for some $(x', x_n) \in \Omega$.

Next we will show that $\varphi = \pm 1$ on the boundary of an appropriate set in $\mathbb{R}^{n-1}$.

LEMMA 3.1. *There exists a bounded, nonempty and open set $\omega \subset \mathbb{R}^{n-1}$ such that $\omega \subset \pi_{n-1}(\Omega)$ and either $\varphi = -1$ on $\partial\omega$ or $\varphi = 1$ on $\partial\omega$.*

PROOF. Suppose that for some $\tilde{x} \in \Omega$ we have $u(\tilde{x}) > 0$. Assuming that $\tilde{x}_n \in (0, 2\pi)$ we find $\varphi(x') > \cos(x_n)$ for $x \in \{\tilde{x}'\} \times [\tilde{x}_n, \pi]$ and $\{\tilde{x}'\} \times [\tilde{x}_n, \pi] \subset \Omega$. Define

$$\omega = \left\{x'; \left(x', \pi\right) \in \Omega\right\}.$$

We have $\tilde{x}' \in \omega$ and since Ω is open and bounded we find that ω is open and bounded. Moreover, if $x' \in \partial\omega$ then $(x', \pi) \in \partial\Omega$ and $\varphi(x') = \cos(\pi) + u(x', \pi) = -1$.

Supposing $u(\tilde{x}) < 0$ for some $\tilde{x} \in \Omega$ and $\tilde{x}_n \in (-\pi, \pi)$ one proceeds by $\omega = \{(x', 0) \in \Omega\}$ to find ω is open and bounded with $\varphi(x') = 1$ on $\partial\omega$. ∎

The following has been proved. There exists a nonempty open cross section ω of Ω parallel to $x_n = 0$ such that

$$\varphi = 1 \text{ on } \partial\omega, \text{ or } \varphi = -1 \text{ on } \partial\omega.$$

In addition to this, as we have seen previously,

$$\begin{cases} -\Delta'\varphi = \gamma\varphi \quad \text{in} \quad \omega, \\ \varphi \in C(\bar{\omega}), \\ \| \varphi \|_\infty \leq 1. \end{cases}$$

When $\varphi = 1$ on $\partial\omega, \varphi$ solves problem (1.5) and the proof of Theorem 1.1 reduces to the proof of Theorem 1.2. By the contrary, if $\varphi = -1$ on $\partial\omega$ it is sufficient to observe that $\widetilde{\varphi} = -\varphi$ satisfies (1.5). We are again in face of a solution of system (1.5).

4. Proof of the $(n-1)$-dimensional result

This section is devoted to the proof of Theorem 1.2. Here and the sequel, to simplify the notation, we denote x' by x and Δ' by Δ. We keep in mind however that, we are in dimension $n-1$, $n \geq 3$.

We first observe that, necessarily, φ changes sign, or, in other words

$$\min_{x\in\bar\omega}\varphi(x)<0. \tag{4.1}$$

Indeed, in order to see (4.1) it is sufficient to observe that, if $\min_{x\in\bar\omega}\varphi(x)\geq 0$, then

$$-\Delta\varphi=\gamma\varphi\geq 0 \text{ in } \omega,$$

since $\gamma\geq 0$. Then, by the maximum principle, the minimum of φ is achieved on $\partial\omega$. Since $\varphi=1$ on $\partial\omega$, this would imply that $\varphi\geq 1$. However, $\|\varphi\|_\infty\leq 1$. Then $\varphi\equiv 1$, which implies $\gamma=0$, as we wanted to prove.

Therefore it is sufficient to consider the case (4.1). We shall see that (4.1) leads to a contradiction. For that, we use a method developed in [1] which is based on the application of the maximum principle to compare φ with suitable reflections and translations of it. In order to illustrate how the method works let us first consider the case where ω is $C^{1,\alpha}$. We then consider the general case.

4.1. The case where $\partial\omega$ is $C^{1,\alpha}$

We will proceed by an argument which is known as a *'sweeping principle'.* A first reference to such an argument is the paper [3].

We set

$$\widetilde\varphi(x)=-\varphi(x) \text{ in } \omega.$$

Given $x_0\in\mathbb{R}^{n-1}$ we define the following translation of $\widetilde\varphi$:

$$\widetilde\varphi_{x_0}(x)=\widetilde\varphi(x-x_0), \text{ in } \omega_{x_0}=\omega+x_0.$$

Obviously, if $|x_0|$ is large enough the graphs of the functions $x\in\omega\to\varphi(x)$ and $x\in\omega_{x_0}\to\widetilde\varphi_{x_0}(x)$ do not intersect.

We now let x_0 tend to $-x_0$ along the segment $I=\{\lambda x_0:-1\leq\lambda\leq 1\}$. In other words we consider the family of functions $\widetilde\varphi_{\lambda x_0}$ defined in $\omega_{\lambda x_0}$.

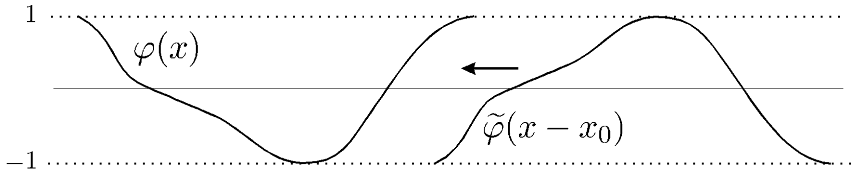

Figure 1

Let λ^* be the first value of the parameter λ so that the graphs of φ and $\widetilde\varphi_{\lambda x_0}$ intersect. The existence of λ^* is guaranteed, for instance, by taking x_0 on the line connecting the origin with one of the points where φ achieves its minimum. In the sequel, to simplify the notation we shall use the notation φ^* and ω^* instead of $\widetilde{\varphi}_{\lambda^* x^0}$ and $\omega_{\lambda^* x_0}$ respectively.

There are two possibilities:

- **Case 1:** Interior contact.

  This is when the graphs of φ and φ^* coincide at a point $x_1 \in \omega \cap \omega^*$ such that x_1 belongs to the interior of $\omega \cap \omega^*$ and therefore $\varphi(x_1) = \varphi^*(x_1) \in (-1, 1)$.

- **Case 2:** Extremal contact.

  We are now in the situation where the graphs of φ and φ^* coincide at a point $x_1 \in \partial(\omega \cap \omega^*)$ where $\varphi(x_1) = \varphi^*(x_1) = -1$. Of course, this second possibility may not arise when $\min \varphi > -1$, but we may not exclude it a priori.

Let us see that in any of these two situations we are led to a contradiction. This will imply that the graphs of φ and φ^* never intersect, which is in contradiction with (4.1). In this way we will deduce that $\gamma = 0$ and $\varphi \equiv 1$.

Case 1: Note that $\varphi \geq \varphi^*$ in the open set $\omega \cap \omega^*$. On the other hand

$$-\Delta(\varphi - \varphi^*) = \gamma(\varphi - \varphi^*) \geq 0 \text{ in } \omega \cap \omega^*. \tag{4.2}$$

Then, by the maximum principle we deduce that either $\varphi - \varphi^* > 0$ in $\omega \cap \omega^*$ or $\varphi - \varphi^* \equiv 0$. Obviously $\varphi - \varphi^* > 0$ in $\omega \cap \omega^*$ may not hold since $[\varphi - \varphi^*](x_1) = 0$. Therefore, necessarily,

$$\varphi \equiv \varphi^* \text{ in } \omega \cap \omega^*. \tag{4.3}$$

Observe however that $\varphi = 1$ in $\partial(\omega \cap \omega^*) \cap \partial\omega$ and that $\varphi^* = -1$ in $\partial(\omega \cap \omega^*) \cap \partial\omega^*$. On the other hand $\partial(\omega \cap \omega^*) \cap \partial\omega \cap \partial\omega^*$ is non empty. Let x_2 be a point in this intersection. We have $\varphi(x_2) = 1$ and $\varphi^*(x_2) = -1$. But, this in contradiction with (4.3).

Case 2. In this second case we also have $\varphi \geq \varphi^*$ in $\omega \cap \omega^*$ and (4.3) holds as well. Then, the argument of Case 1 shows that $\varphi > \varphi^*$ in $\omega \cap \omega^*$. On the other hand, we know that for $x_1 \in \partial\left(\omega \cap \omega^*\right)$, $\varphi(x_1) = \varphi^*(x_1)$. By Hopf's maximum principle we deduce that $\partial(\varphi - \varphi^*)(x_1)/\partial\nu < 0$, ν being the outward unit normal to $\omega \cap \omega^*$ in x_1. Assume for instance that $\varphi(x_1) = -1$. Then, taking into account that φ is smooth in the interior of ω and that it achieves its minimum in x_1 we deduce that $\nabla\varphi(x_1) = 0$. Therefore $\partial\varphi^*(x_1)/\partial\nu > 0$. But then, in a neighborhood of x_1 in ω^*, we deduce that

$$\varphi^*(x) \geq -1 + cd(x, \partial\omega^*) \tag{4.4}$$

for a suitable positive constant $c > 0$, where $d(\cdot, \partial\omega^*)$ denotes the distance to the boundary $\partial\omega^*$.

In view of (4.4) and taking into account that $\nabla\varphi(x_1) = 0$, $\varphi(x_1) = -1$, we immediately deduce that the intersection of the graphs of φ and φ^* occurs before the parameter λ reaches the value λ^*, which is in contradiction with the definition of λ^* itself.

The proof is the same when $\varphi(x_1) = \varphi^*(x_1) = 1$.

Note that in Case 2 the regularity of the domain has been used to guarantee the applicability of Hopf's maximum principle.

Note also that both in Case 1 and Case 2 the point of intersection is not necessarily unique. As we have seen, in order to reach the contradiction it is sufficient that the contact arises.

4.2. General ω

In order to avoid the difficulties related to the application of Hopf's maximum principle we are going to introduce a radially symmetric function that will play the role of φ in the arguments above.

We assume that $0 \in \omega$ and let B the smallest ball of $\mathbb{R}^{n-1}$ having 0 as center containing ω. We introduce the functions

$$\varphi^R(x) = \varphi(Rx),\ \forall x \in \omega^R = R^{-1}\omega, \tag{4.5}$$

for any rotation R in $\mathbb{R}^{n-1}$.

We then set

$$\phi(x) = \min_{R=\text{rotation}} \varphi^R(x),\ \forall x \in B \tag{4.6}$$

The function ϕ, the minimum of all the rotated functions φ^R, is radially symmetric and it is defined on B. Note that, for any $x \in B$ the minimum in (4.6) has to be taken over the rotations R such that $Rx \in \omega$.

Taking into account that φ^R solves

$$-\Delta\varphi^R = \gamma\varphi^R \text{ in } \omega^R$$

for all rotation R, one deduces as in [1] that ϕ is a supersolution in distributional sense of the same problem in B:

$$\int_B (-\Delta\psi\,\phi - \gamma\psi\,\phi)\,dx \geq 0 \text{ for all } \psi \in \mathcal{D}^+(B), \tag{4.7}$$

where $\mathcal{D}^+(B) = \{\psi \in C_0^\infty(B)\,;\psi \geq 0\}$. Note also that

$$\begin{cases} \phi \in C(\bar{B}), \\ \phi = 1 \text{ on } \partial B, \\ \|\phi\|_\infty \leq 1. \end{cases} \tag{4.8}$$

Thus, ϕ fulfils the same conditions of (1.5) except that the equation satisfied by φ has to be replaced by the inequality (4.7).

We now apply the argument of the previous section to ϕ. Again let ϕ^* being the reflected and translated function of ϕ that intersects ϕ for the first time, say at $x_1 \in \bar{B} \cap \bar{B}^*$. We have the following

$$\begin{cases} \phi - \phi^* \in C\left(\bar{B} \cap \bar{B}^*\right), \\ \displaystyle\int_{B \cap B^*} (-\Delta\psi - \gamma\psi)\,(\phi - \phi^*)\ dx \geq 0 \quad \text{for all } \psi \in \mathcal{D}^+(B \cap B^*), \\ \phi - \phi^* \geq 0 \qquad\qquad \text{in } B \cap B^*, \\ (\phi - \phi^*)(x_1) = 0 \qquad \text{for some } x_1 \in \partial\,(B \cap B^*). \end{cases} \tag{4.9}$$

As in [1] we may conclude that either $\phi - \phi^* > 0$ in $\operatorname{int}(B \cap B^*)$ or $\phi - \phi^* \equiv 0$ in $\bar{B} \cap \bar{B}^*$. Notice that we have

$$\phi(x) - \phi^*(x) = 2 \text{ for } x \in \partial B \cap \partial B^*.$$

Since $\partial B \cap \partial B^*$ is nonempty $(n-1 > 1)$ one finds that ϕ and ϕ^* cannot be identical.

We again distinguish the two cases:

- **Case 1:** Interior contact, $x_1 \in \operatorname{int}(B \cap B^*)$,
- **Case 2:** Extremal contact, $x_1 \in (\partial B \cap B^*) \cup (B \cap \partial B^*)$.

Let us analyze both cases:

Case 1: The strict inequality in the interior contradicts the assumption $x_1 \in \operatorname{int}(B \cap B^*)$.

Case 2: In this case we do not have enough regularity of ϕ and ϕ^* to apply Hopf's Lemma directly. Let us consider, for instance, the case where

$$\phi(x_1) = \phi^*(x_1) = -1,$$

hence $x_1 \in \partial B^*$. Note that ϕ achieves its minimum in x_1. Then for some rotation R_1 the function φ^{R_1} achieves its minimum as well at x_1. This implies, taking into account that φ^{R_1} is smooth in x_1, which necessarily lies in the interior of ω, that

$$-1 \leq \varphi^{R_1} \leq -1 + c \mid x - x_1 \mid^2$$

in a neighborhood $\mathcal{N}_{x_1}$ of x_1 for a suitable $c > 0$. We immediately deduce that ϕ verifies the same condition, i.e. in the neighborhood $\mathcal{N}_{x_1}$ of x_1, ϕ satisfies

$$-1 \leq \min_{R=\text{rotation}} \phi(x) \leq \varphi^{R_1}(x) \leq -1 + c \mid x - x_1 \mid^2,\ \forall x \in \mathcal{N}_{x_1}. \tag{4.10}$$

Now the strong maximum principle implies that

$$\phi - \phi^* \geq c' \Phi_1 \text{ in } B \cap B^* \tag{4.11}$$

for a suitable $c' > 0$, Φ_1 being the first eigenfunction of $-\Delta$ in $H_0^1(B \cap B^*)$. Taking into account that $\partial(B \cap B^*)$ is smooth at x_1, we also know that

$$\Phi_1 (x) \geq c'' d(x, \partial(B \cap B^*)), \forall x \in \mathcal{N}_{x_1} \tag{4.12}$$

for a suitable $c'' > 0$.

Combining (4.10), (4.11) and (4.12) we obtain immediately a contradiction as in the previous section.

Remark: It is interesting to analyze why the proof of Theorem 1.2 does not work when $n = 2$. The same ideas can be used. We then reach the conclusion that $\varphi \equiv \varphi^*$. However, when $n = 2$, $\omega \cap \omega^*$ is just an interval, whose boundary is constituted by two points. At the left end both functions φ and φ^* take value -1, while, at the right end, they take the value 1. There is no contradiction since these two extremes are isolated. The situation is completely different when $n \geq 3$ since $\partial(\omega \cap \omega^*)$ contains points where simultaneously $\varphi = 1$ while $\varphi^* = -1$. In fact, when $n = 2$, due to the symmetry of the function $\varphi(x_1) = \cos(2\pi x_1)$ in the interval $\omega = (-1, 0)$, it is easy to see that $\varphi = \varphi^*$ in $(-1/2, 0)$ without contradiction. ■

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