



URJC, Diciembre 06.

Control, Optimal Design and Numerics

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DOMINO: Desarrollo de aplicación informática para el Diseño Óptimo aeronáutico Mediante técnicas NOvedosas

<http://www.dominoproject.com>

SIMUMAT: Modelización Matemática y Simulación Numérica en Ciencia y Tecnología

<http://www.simumat.es/ESP/>

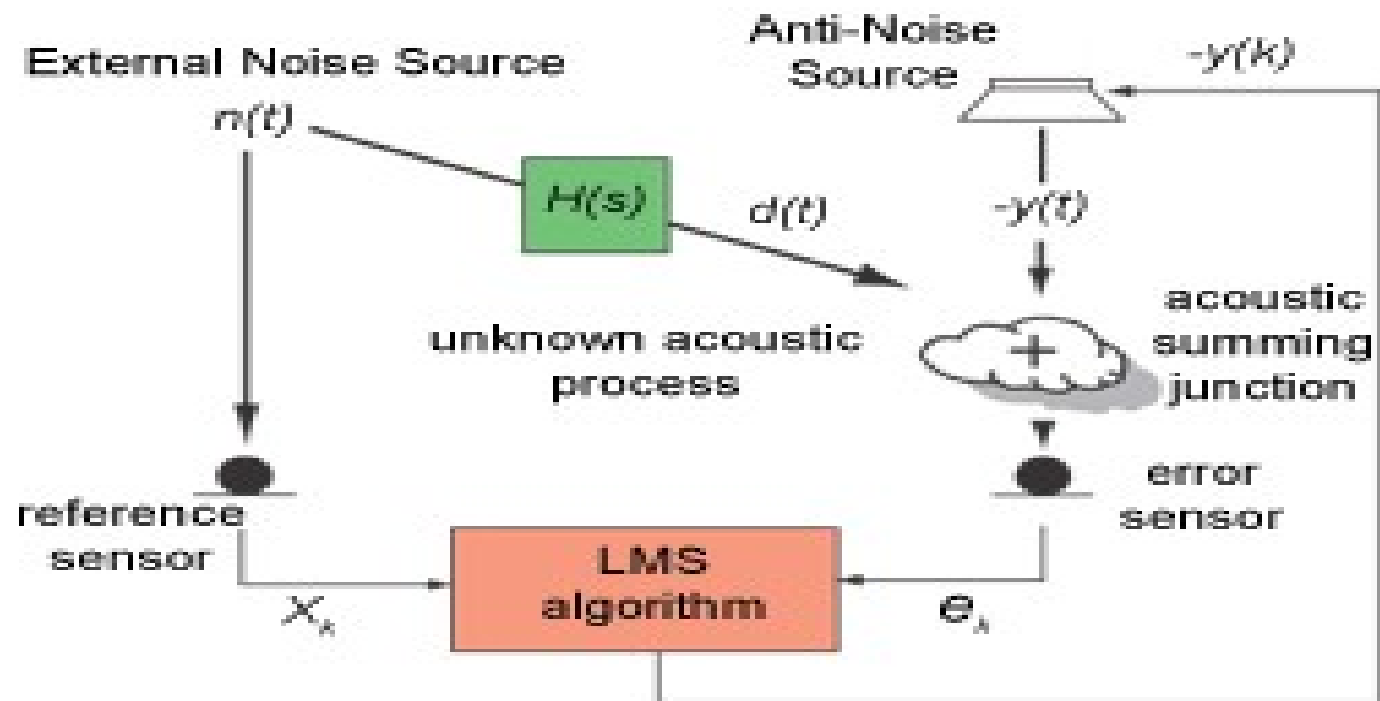
IMDEA: Instituto Madrileño de Estudios Avanzados - Matemáticas

<http://www.imdea.org/>

MOTIVATION / A FEW EXAMPLES:

- Noise reduction in cavities and vehicles.

Typically, the models involve the wave equation for the **acoustic waves** coupled with some other equations modelling the **dynamics of the boundary structure**, the action of **actuators**, possibly through **smart mechanisms** and materials.



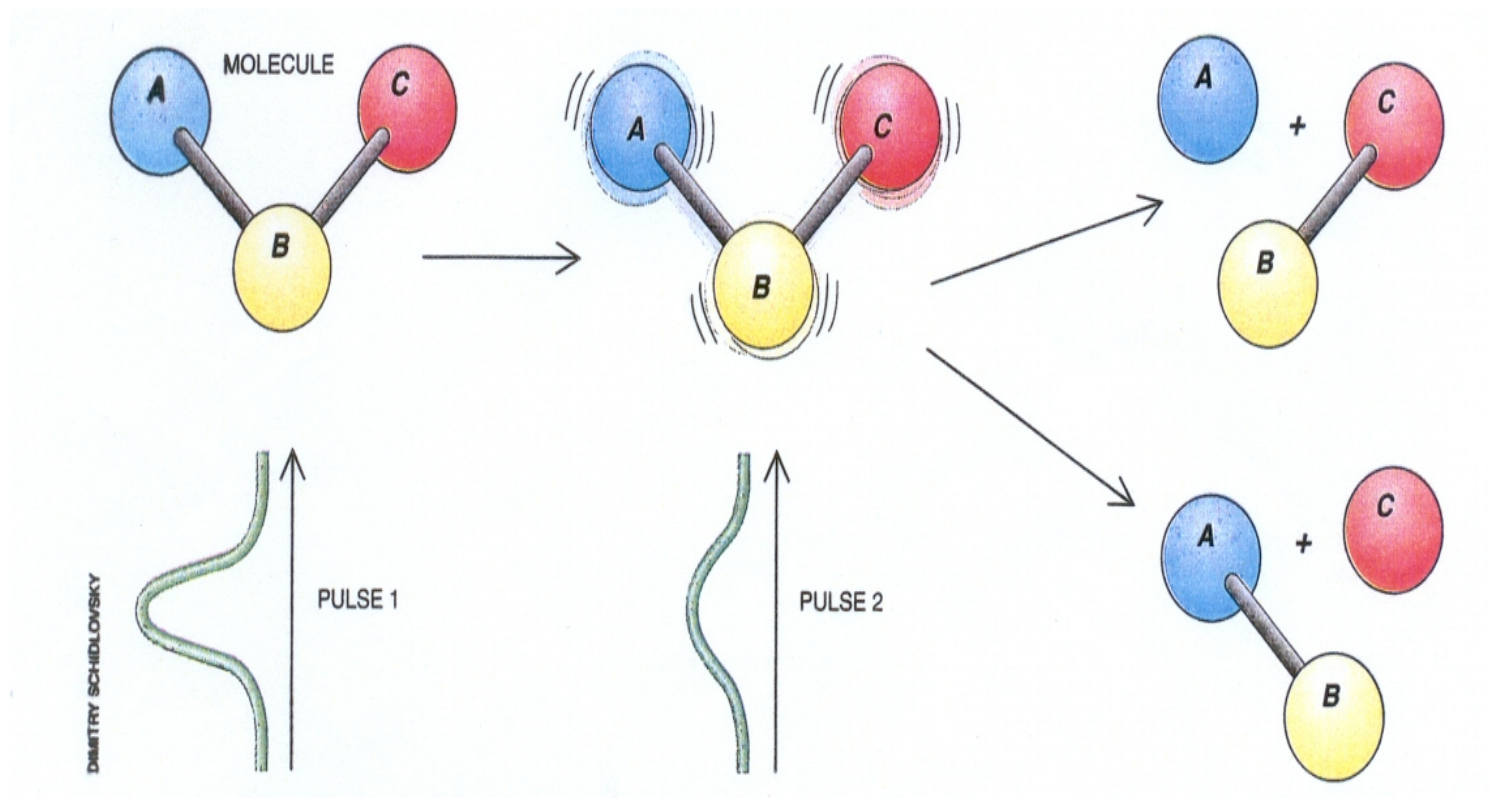
http://www.ind.rwth-aachen.de/research/noise_reduction.html

Noise reduction is a subject to research in many different fields. Depending on the environment, the application, the source signals, the noise, and so on, the solutions look very different. Here we consider noise reduction for audio signals, especially speech signals, and concentrate on common acoustic environments such as an office room or inside a car. The goal of the noise reduction is to reduce the noise level without distorting the speech, thus reduce the stress on the listener and - ideally - increase intelligibility.

- Quantum control and Computing.

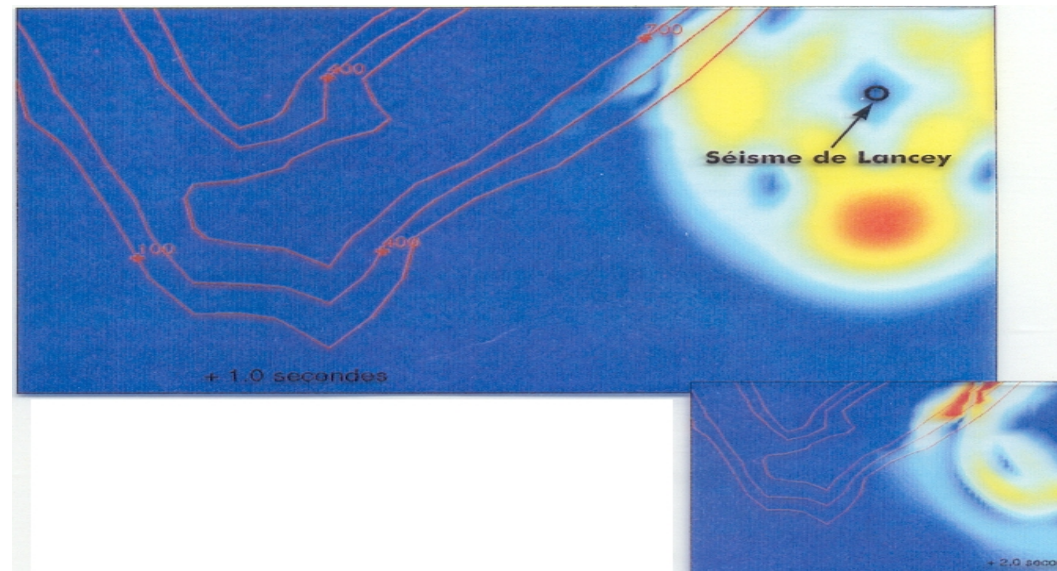
Laser control in Quantum mechanical and molecular systems to design **coherent vibrational states**.

In this case the fundamental equation is the **Schrödinger equation** that may be viewed as a **wave equation with infinite speed of propagation**. The laser beam interacts with the Schrödinger free dynamics:



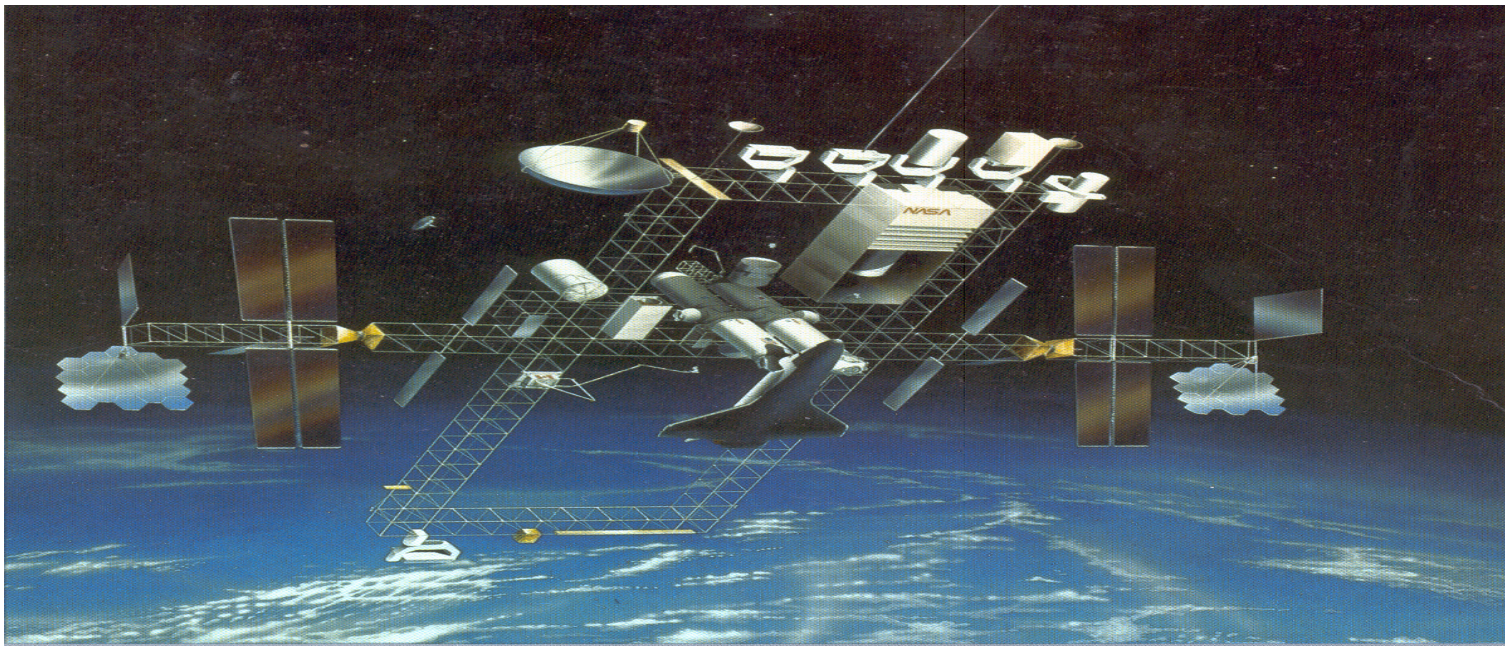
P. Brumer and M. Shapiro, Laser Control of Chemical reactions, Scientific American, March, 1995, pp.34-39.

- **Seismic waves, earthquakes:** The interaction of seismic waves with buildings...



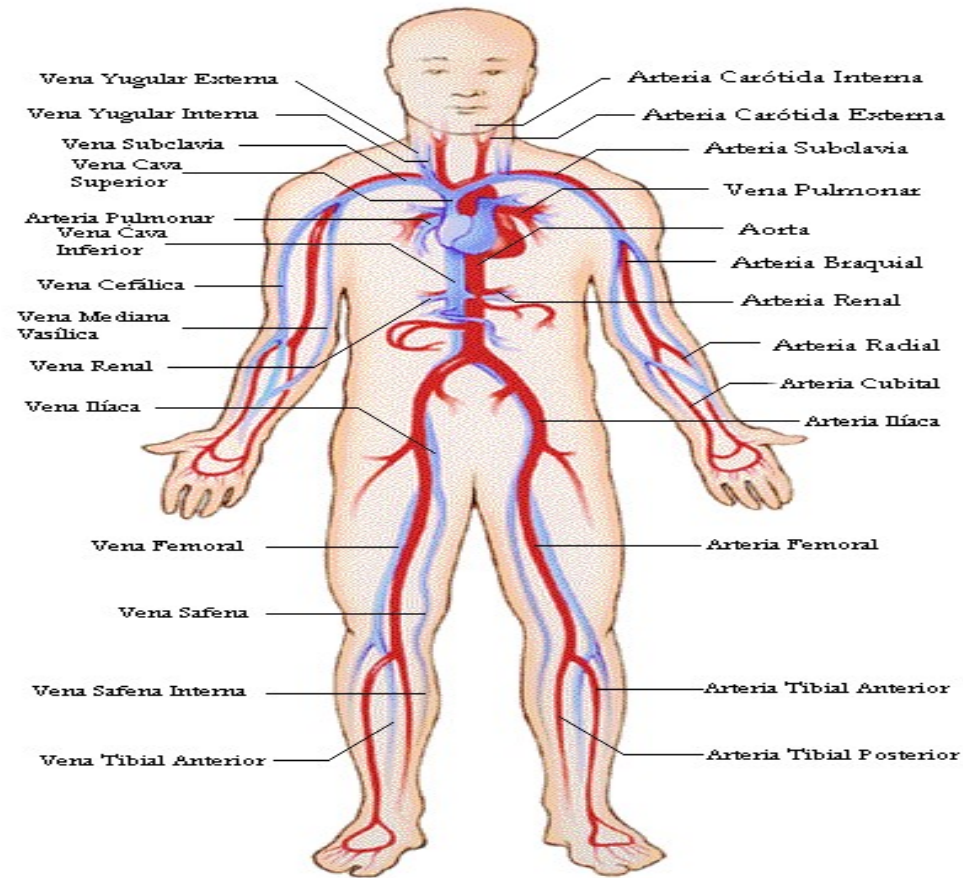
F. Cotton, P.-Y. Bard, C. Berge et D. Hatzfeld, Qu'est-ce qui fait vibrer Grenoble?, La Recherche, 320, Mai, 1999, 39-43.

- Flexible structures.

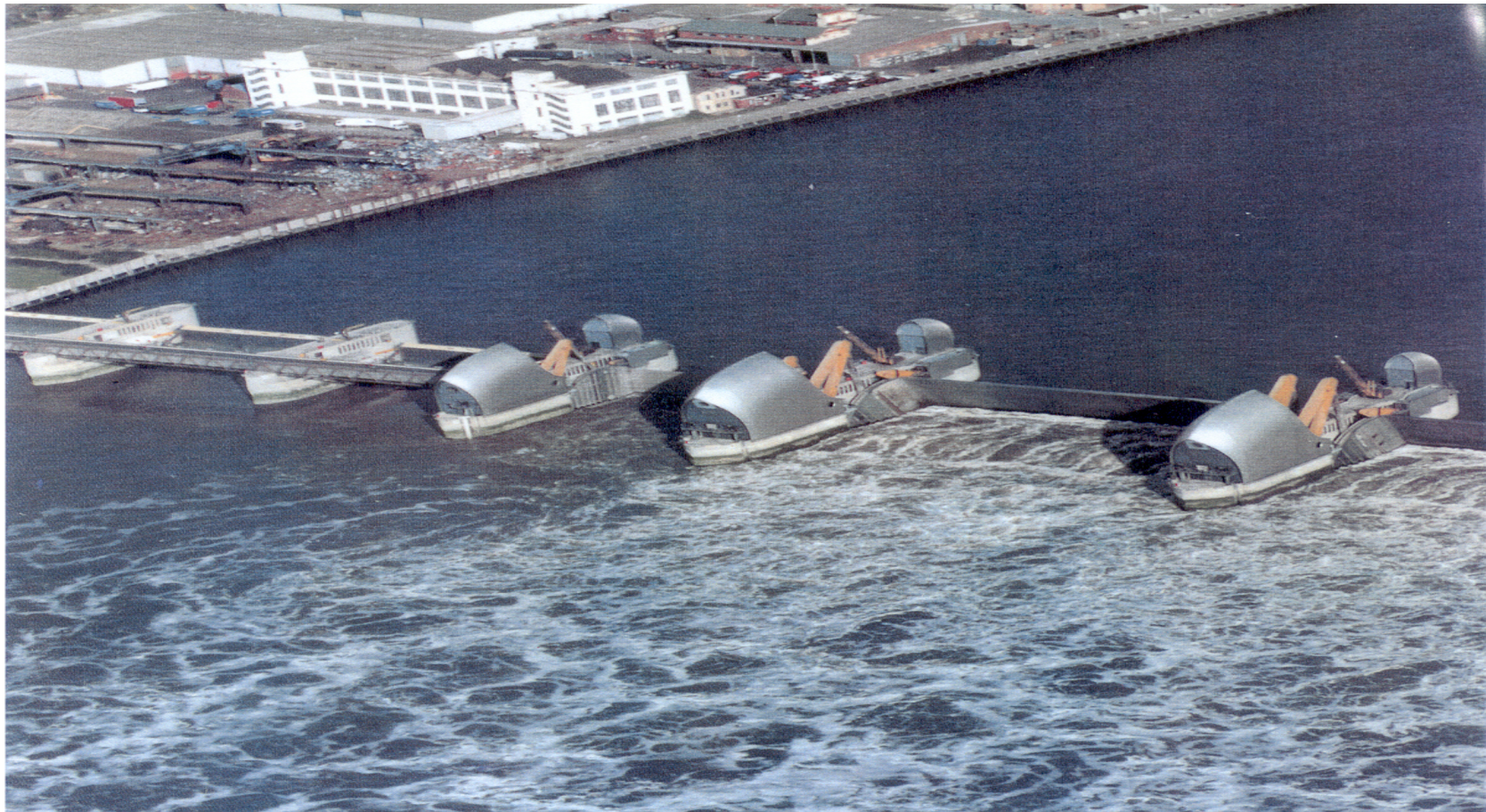


SIAM Report on "Future Directions in Control Theory. A Mathematical Perspective", W. H. Fleming, Chairman, 1988.

And many others...



The human cardiovascular system: Blood in arteries.



The Tames barrier

KEY WORDS AND IDEAS:

CONTROL

~ OPTIMIZATION

~ OPTIMAL DESIGN

Optimal design or optimal shape design = Control problem in which the control variable is the shape of the domain where the dynamics under consideration evolves.

MAIN STRATEGY:

Use a **descent method** for efficiently reaching the minimum.

For that, an efficient **computation of gradients** is needed.

When the deformations under consideration involve many variables (shape functions) the gradient is constituted by very many directional derivatives.

The short-cut: **The adjoint state!**

The adjoint system **looks** very much like the state equation. It provides all directional derivatives and accelerates the efficiency of the descent method.

TWO APPROACHES

- * PDE approach:

PDE \rightarrow Control Theory for PDE \rightarrow Continuous Adjoint

\rightarrow Numerical approximation of controls.

- * Finite-dimensional approach:

PDE \rightarrow numerical approximation of PDE \rightarrow Discrete Adjoint

\rightarrow control for the numerical scheme/finite dimensional model.

COMPARISON:

THE PDE APPROACH:

Advantages:

- It is more consistent with the developments of Continuum Mechanics. It uses really PDE modelling.
- There is a well established theory of PDE's and also of numerical approximation schemes.

The drawbacks:

- The existing theory is often insufficient to deal with realistic and complex models;
- It requires a master of various sophisticated tools: Functional Analysis, Geometry, Asymptotics of Solutions, Numerical Analysis,.....
- Numerical analysis is required twice: One to approximate the state equations and another one for the adjoint ones.

THE FINITE-DIMENSIONAL APPROACH:

Advantages:

- It requires less sophisticated mathematical tools.
- It is closer to the effective computer programming.

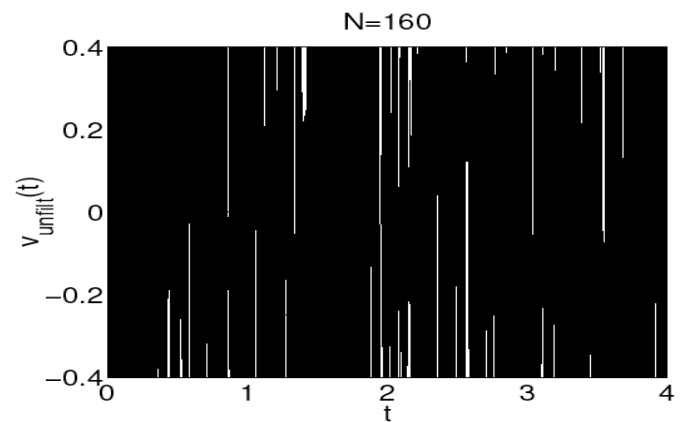
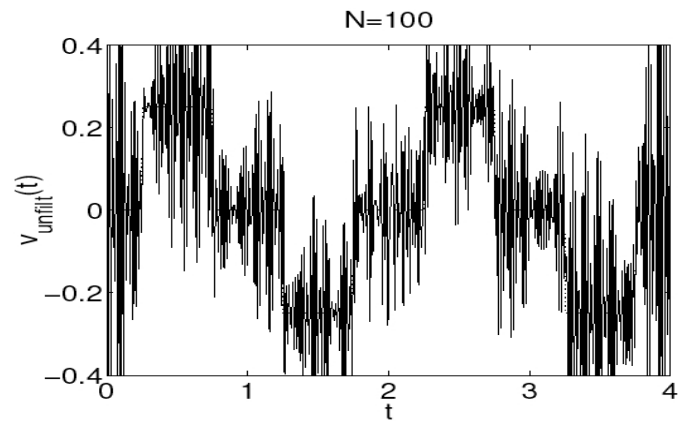
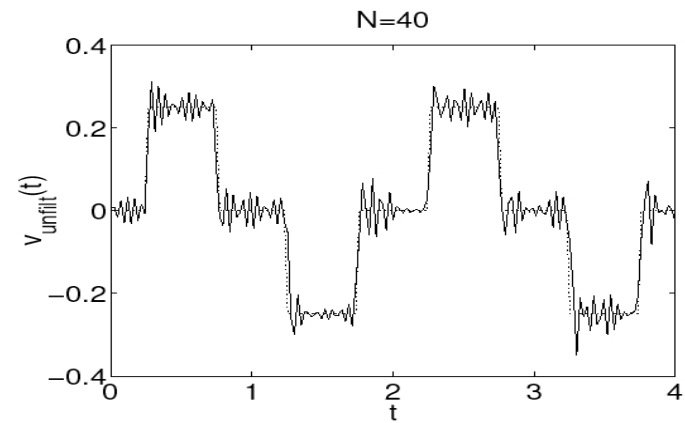
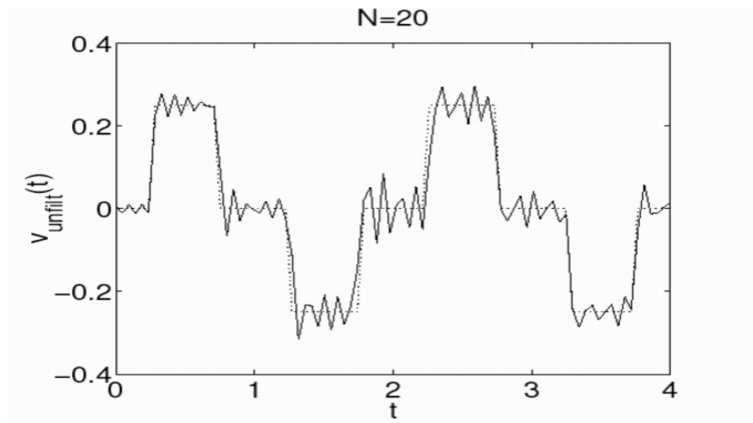
The drawbacks:

- Many numerical schemes are not differentiable (“if conditions” for upwinding and viscosity).
- It may ignore some subtle and relevant aspects on the behavior of solutions;
- Numerical schemes that are stable for solving IVP may be unstable for control.
- One may lose intuition about the behavior and nature of the adjoint state.

DO BOTH APPROACHES GIVE THE SAME RESULTS?

DO THE PROCESSES OF CONTROL AND NUMERICS COMMUTE?

IN GENERAL THEY DO NOT, THIS IS PARTICULARLY THE CASE FOR WAVE-LIKE PHENOMENA.



WHY? SPURIOUS NUMERICAL SOLUTIONS!

THE SOLUTION?

A CLEVER COMBINATION OF BOTH CONTINUOUS AND DIS-
CRETE APPROACHES.



DOMINO Project: AIRBUS-Spain, INTA (Instituto Nacional de Técnicas Aeronáuticas), UAM.

Visión de la Aeronáutica en el año 2020



Los retos

Los métodos

Las herramientas

Un ejemplo

Conclusiones

Quality & Affordability

- | Permanent trend monitoring
- | Flexible cabin environments
- | Passenger services
- | Anticipatory maintenance systems
- | Integrated avionics
- | ATM related airborne systems
- | Novel materials and structural concepts
- | Lead time reductions
- | Integrated design manufacturing and maintenance systems
- | Advanced design methods
- | System validation through modelling and simulation
- | Concurrent engineering



Environment

- | Drag reduction through conventional and novel shapes
- | Fuel additives
- | Noise reduction
- | Propulsion concepts
- | Emission reduction
- | Environmentally friendly production, maintenance and disposal
- | Better aircraft/engine integration



Safety

- | Flight hazard protection
- | Advanced avionics
- | Probability and risk analysis
- | Computational methods
- | Human error checking systems

El diseño óptimo aeronáutico



DOMINÓ Project
AIRBUS-EPITA/UMH/UPV

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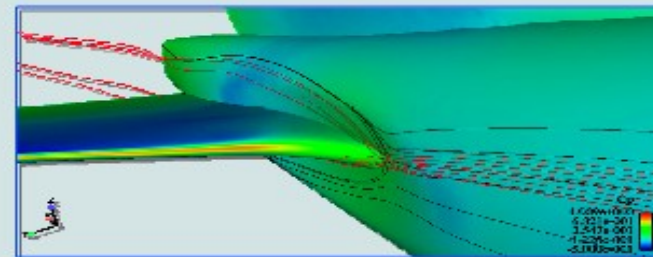
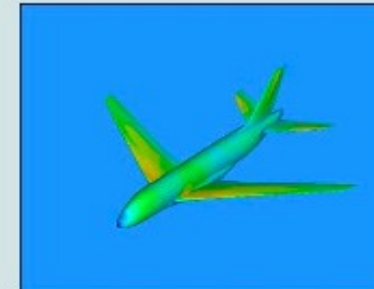
La optimización es el objetivo final de todos los proyectos prácticos de diseño

Numerosas variables
+ Experimentos caros
Intuición falla

Es necesaria una aproximación sistemática al problema.

¿Cuál es la mejor aeronave?

- ¿Qué significa "lo mejor posible"?
- ¿Cómo estimar las características de diseños para poder comparar?
- ¿Cómo elegir las variables del diseño mejores?



La optimización numérica... ¿qué falla?



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- El análisis de una configuración completa del avión es algo muy complejo.
- Cualquier análisis hace ciertas aproximaciones y deja ciertas cosas fuera. Los optimizadores son especialistas explotar cualquier debilidad en el análisis.
- Un optimizador se limita a considerar diseños que sean descritos por el sistema de parámetros seleccionado.
- Una función objetivo puede variar de una manera no suave.
- El tiempo de cálculo requerido en ciertas aplicaciones es prohibitivo

Una solución matemática: la teoría de control



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La teoría de control aplicada al diseño aerodinámico permite:

- Diseño con **gran número de variables**
- Cálculo de la **sensibilidad de un funcional**

Control activo	10%
Materiales	20%
Flujo Laminar	10%
Mejoras en el ala	10%
Propulsión	20%
Total	70%



+



=



La metodología **Adjunta Continua y Discreta** = método muy eficiente de calcular los **gradientes de un funcional J** que depende de la solución a través de un sistema de variables de diseño.



1er Paso: Ingeniería. Aplicar tecnología para resolver el problema.

2º Paso: Matemático. Si soluciones regulares resueltas con esquemas diferenciables: Resolver con técnicas más sencillas.

Visión inicial de la ingeniería



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Dominio fluido Ω delimitado por un campo lejano Γ y varios contornos de pared sólida S .

$$J(S^{\min}) = \min_{S \in \mathcal{S}_{ref}} J(S).$$

Objeto minimizar un determinado funcional J de las variables de flujo U

$$J(U) = \int_S j(U) ds.$$

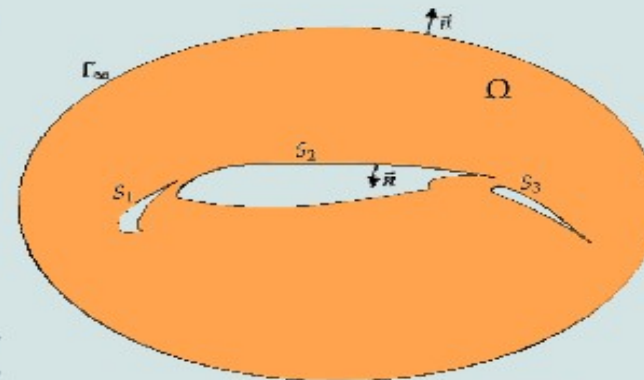
Marco continuo, asumiendo una solución regular U , la variación del funcional J se puede evaluar como:

$$\delta J(U) = \int_{\partial S} j(U) ds + \int_S j'(U) \delta U ds.$$

1^{er} término debido a deformación del contorno.

2^o término variación infinitesimal en la solución.

$$\int_{\partial S} j(U) ds = \int_S (\partial_n j - \kappa j) \delta S ds,$$

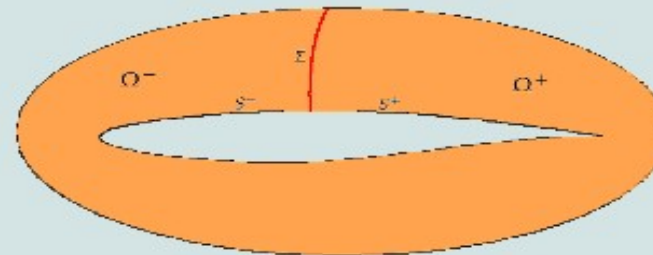


Una **discontinuidad debe ser considerada a lo largo de una curva regular Σ** . La curva Σ divide el fluido en dos subdominios Ω^- and Ω^+ (a la izquierda y derecha de Σ , respectivamente).

El **calculo clásico de la derivada** es modificado para **incluir el efecto de las sensibilidad de la posición del choque** con respecto de deformaciones en la superficie.

$$\delta J = \int_{\partial\Omega} f(U) ds + \int_{S^-\cup S^+} f'(U) \delta U ds + [f(U)]_{\Sigma} \delta\Sigma,$$

El último término: influencia de la **variación en la posición de la discontinuidad**.



Concepto ingenieril de la optimización



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AIRBUS-INTA/UMH/UPV

Los retos

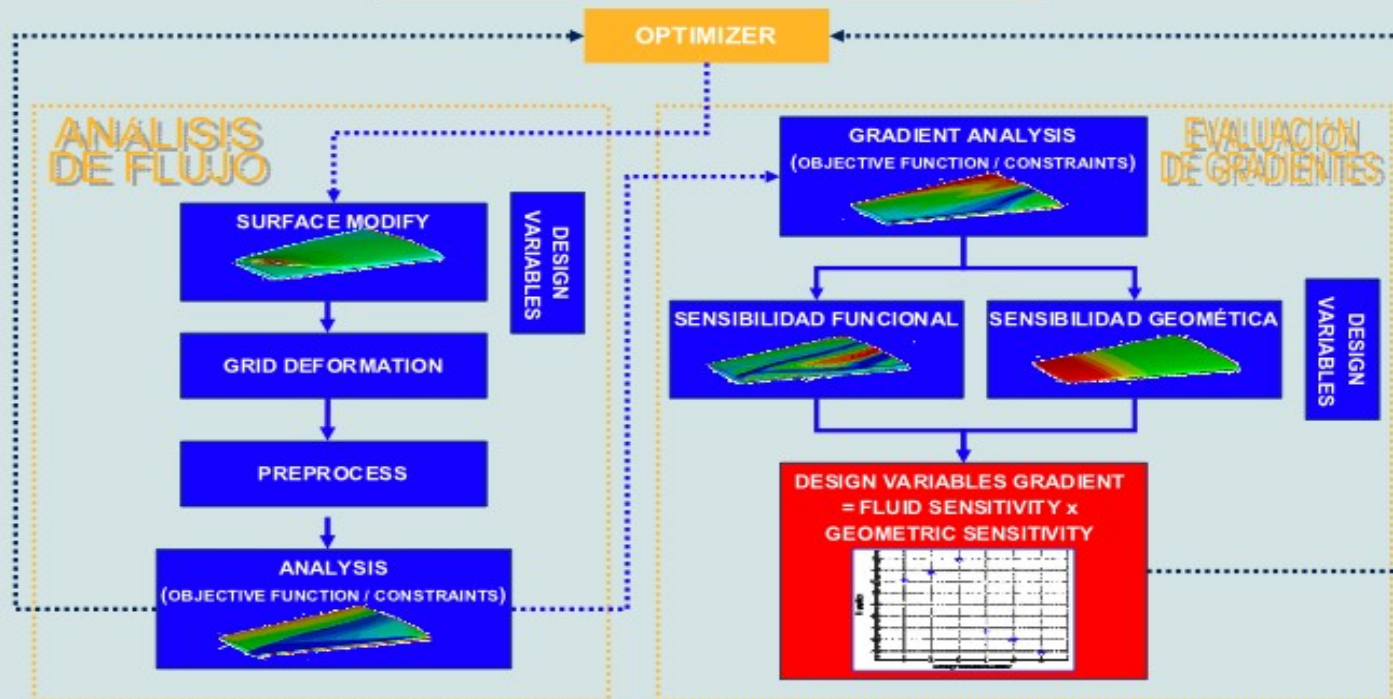
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PROBLEMA DE DISEÑO
CONDICIONES DE FLUJO & VARIABLES DE DISEÑO

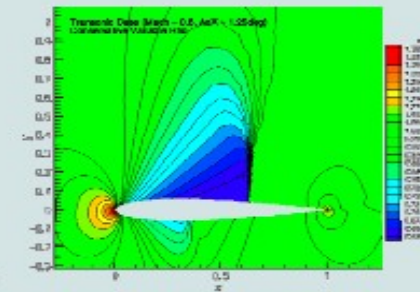


Ecuaciones de Euler + relaciones de Rankine-Hugoniot

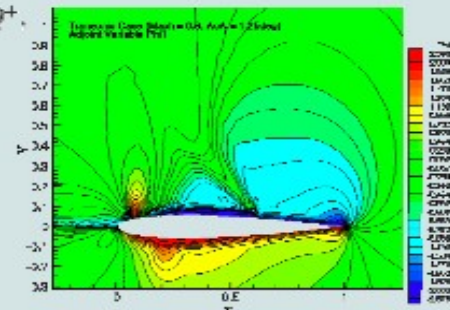
$$\begin{cases} \partial_t U + \nabla \cdot \vec{F} = 0, \\ \vec{v} \cdot \vec{n}|_S = 0, \end{cases} \quad [\vec{F} \cdot \vec{n}_\Sigma]_\Sigma - s[U]_\Sigma = 0,$$

La variación del funcional se puede calcular como

$$\delta J = \int_{\delta S} j(U) dt - \int_{S \cup S^+} (\vec{n}_S \cdot \delta \vec{v}) (\rho \psi_1 + \rho \vec{v} \cdot \vec{\Phi} + p H \psi_2) dt - [j(U)]_{x_0} (\vec{n}_S \cdot \vec{n}_\Sigma) \delta S(x_0),$$



$$\begin{cases} \left(\frac{\partial \vec{F}}{\partial U}\right)^T \cdot \vec{v} \Psi = 0, & \text{in } \mathcal{Q}^- \cup \mathcal{Q}^+, \\ |\Psi|_\Sigma = 0, & \text{on } \Sigma, \\ ([\rho \vec{v}]_\Sigma \cdot \vec{r}_\Sigma) (\partial_{t_2} \Psi_1 + H \partial_{t_2} \Psi_2) \\ + [\rho (\vec{v})^2 + 2P]_\Sigma \vec{r}_\Sigma \cdot \partial_{t_2} \vec{\Phi} = 0, & \text{on } \Sigma, \\ \Psi \left(\vec{n}_{T_\infty} \cdot \frac{\partial \vec{F}}{\partial U} \right) \delta U = 0, & \text{on } \Gamma_\infty, \\ \vec{\Phi} \cdot \vec{n}_S - j'(P), & \text{on } S, \\ \Psi^T(x_0) \left[\vec{F} \cdot \vec{r}_\Sigma \right]_{x_0} = [j(P)]_{x_0} & \text{at } x_0, \\ \Psi^T(x_0) \left[\vec{F} \cdot \vec{r}_\Sigma \right]_{x_0} = 0, & \text{at } x_{0-}. \end{cases}$$



Comprensión a través de la ecuación de Burgers

$-\partial_x p - u \partial_x p = 0$
 $p(x, T) = f(u(x, T))$
 $[p]_{x(t)} = 0,$
 $\partial_{x_0}(p) = 0$

$\partial_t u + \partial_x \left(\frac{u^2}{2}\right) = 0$
 $u(x, 0) = u^0$
 $\mathcal{L}'(t)[u]_{x(t)} = \left[\frac{u^2}{2}\right]$

Ecuaciones adjuntas de Navier-Stokes



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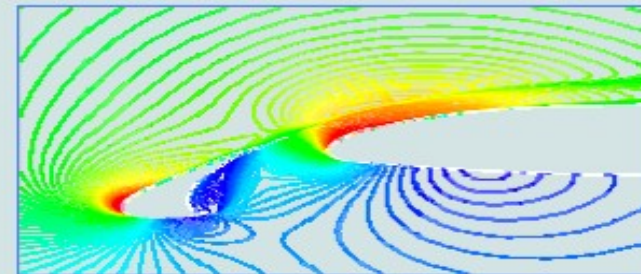
Las ecuaciones de Navier-Stokes

$$\begin{cases} \partial_t U + \nabla \cdot (\mathcal{F} + \mathcal{F}^r) = 0, \\ \vec{r}|_S = 0, \\ \partial_n T|_S = 0, \end{cases}$$

La ecuación adjunta es

$$-(\vec{\lambda} - \vec{\lambda}^r)^T \cdot \nabla \Psi + \nabla \cdot \left(\begin{pmatrix} D_x^T & D_y^T \\ D_x^r & D_y^r \end{pmatrix} \cdot \begin{pmatrix} \partial_x \Psi \\ \partial_y \Psi \end{pmatrix} \right) = 0$$

$$\frac{\partial \mathcal{F}}{\partial U} = \vec{\lambda}, \quad \frac{\partial \mathcal{F}^r}{\partial U} = \vec{\lambda}^r, \quad \frac{\partial F_x^r}{\partial \left(\frac{\partial U}{\partial x} \right)} = D_{xxx}, \quad \frac{\partial F_x^r}{\partial \left(\frac{\partial U}{\partial y} \right)} = D_{xyx}, \quad \frac{\partial F_y^r}{\partial \left(\frac{\partial U}{\partial x} \right)} = D_{xyy}, \quad \frac{\partial F_y^r}{\partial \left(\frac{\partial U}{\partial y} \right)} = D_{yyy}.$$



La variación del funcional se calcula como

$$\delta J = \delta \int_S j(\vec{f}, T) dS = \int_{\delta S} j(\vec{f}, T) ds - I_{\text{ext}}, \quad \vec{f} = P\vec{n}_S - \vec{n}_S \cdot \sigma.$$

$$\begin{cases} \Psi_2 = \frac{\partial j}{\partial T}, \quad \Psi_3 = \frac{\partial j}{\partial \vec{f}}, \quad k \partial_n \Psi_4 = \frac{\partial j}{\partial T}, \\ I_{\text{ext}} = \int_S \vec{n}_S \cdot \delta \sigma (\rho \psi_1 + \rho H \psi_4) - \psi_4 \vec{n}_S \cdot \sigma \cdot \delta \sigma + \vec{n}_S \cdot \Sigma \cdot \delta \sigma ds \\ \quad - \int_S k \psi_4 \partial_n (\delta T) ds, \\ \delta u|_S = -\delta S \partial_p u, \\ \delta v|_S = -\delta S \partial_k v, \\ \vec{n}_S \cdot \nabla \delta T|_S = -\delta S \partial_n (\nabla T) \cdot \vec{n} + (\partial_s \delta S) (\nabla T) \cdot \vec{t}_S. \end{cases}$$

Problema matemático con la 2ª derivada

Test y validaciones ingenieriles



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Los retos

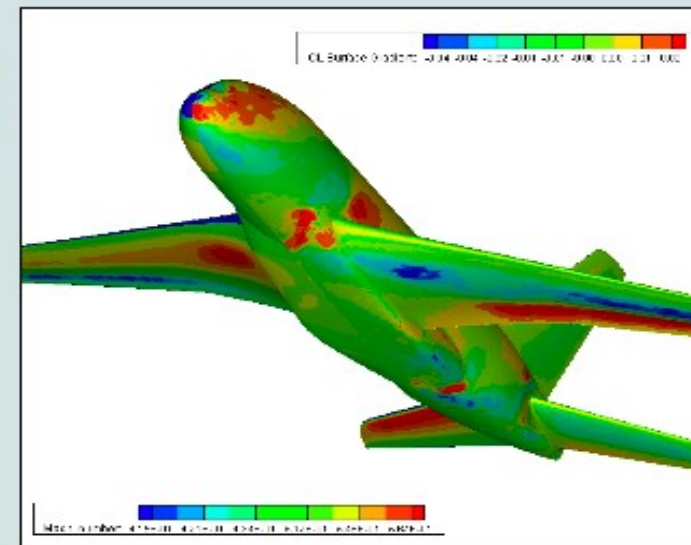
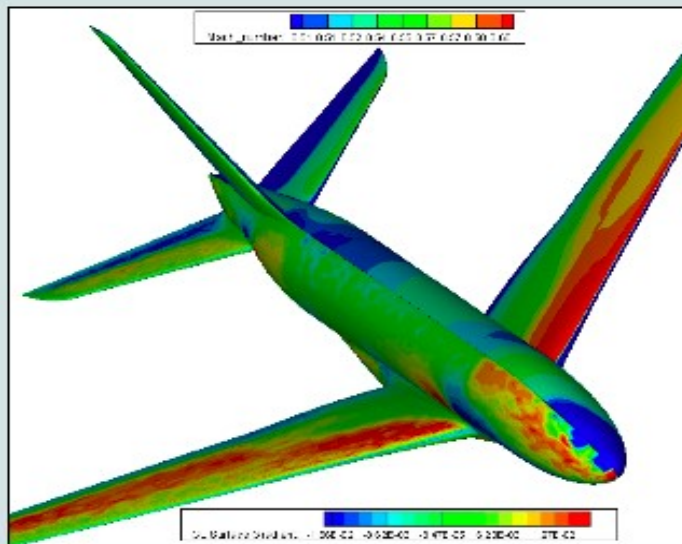
Los métodos

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Un ejemplo

Conclusiones

Euler computation, AoA = 0.0°, Mach 0.85



Test y validaciones ingenieriles



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Los retos

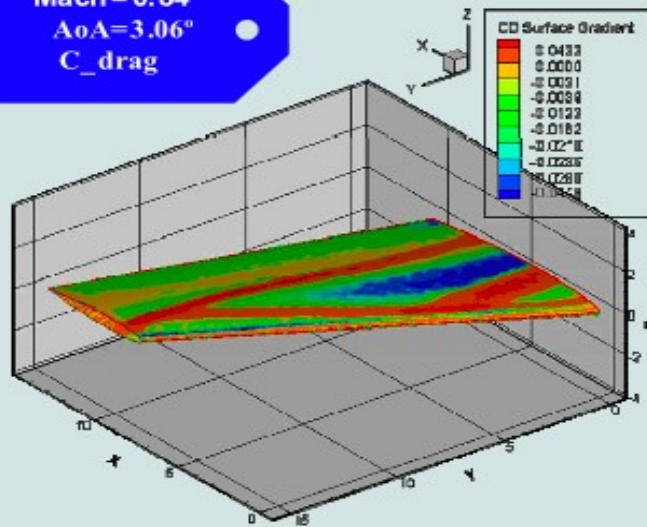
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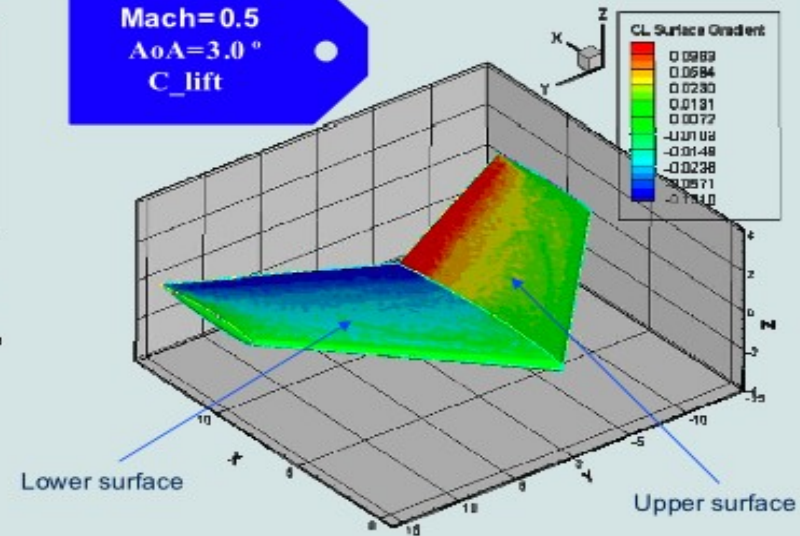
Conclusiones

Mach = 0.84
AoA = 3.06°
C_d drag

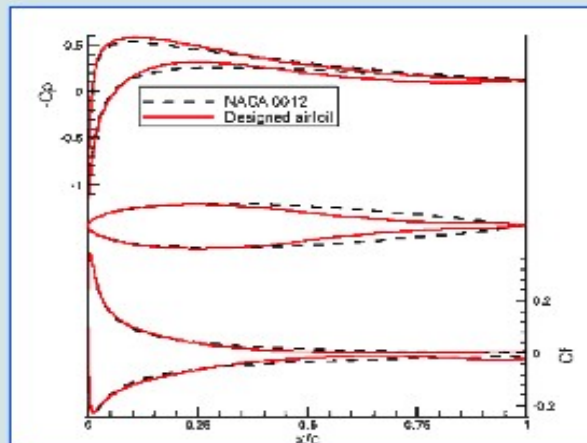


Sensibilidad C_D ONERA M6

Mach = 0.5
AoA = 3.0°
C_l lift

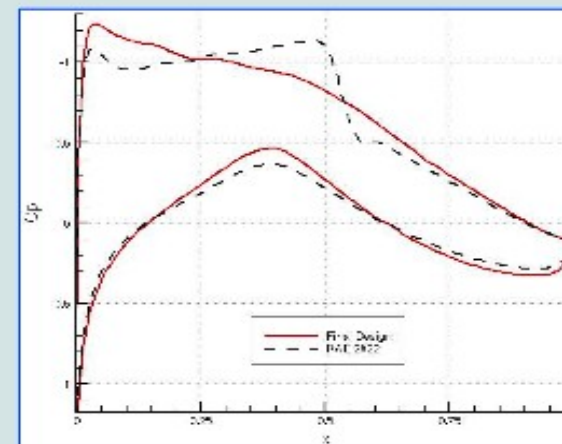


Sensibilidad C_L ONERA M6



Reducción de resistencia
(régimen laminar)

Ángulo de Ataque = 2.50°
Número de Mach = 0,3
Número de Reynolds 1000
Restricciones Geométricas



Reducción de resistencia
(régimen turbulento)

Ángulo de Ataque = 2.54°
Número de Mach = 0,734
Número de Reynolds $6.5e6$
Restricciones Geométricas

EN EL CAMPO DE LA OPTIMIZACIÓN

- Cálculo de la segunda derivada del funcional
- Desarrollo de sistemas expertos para mejorar la convergencia de la optimización.
- Multi-Objetivo.
- Desarrollo de buenas parametrizaciones.
- Introducción in situ de la sensibilidad del choque.

EN EL CAMPO DE COMCEPTOS FUTUROS DE SIMULACIÓN Y ANÁLISIS

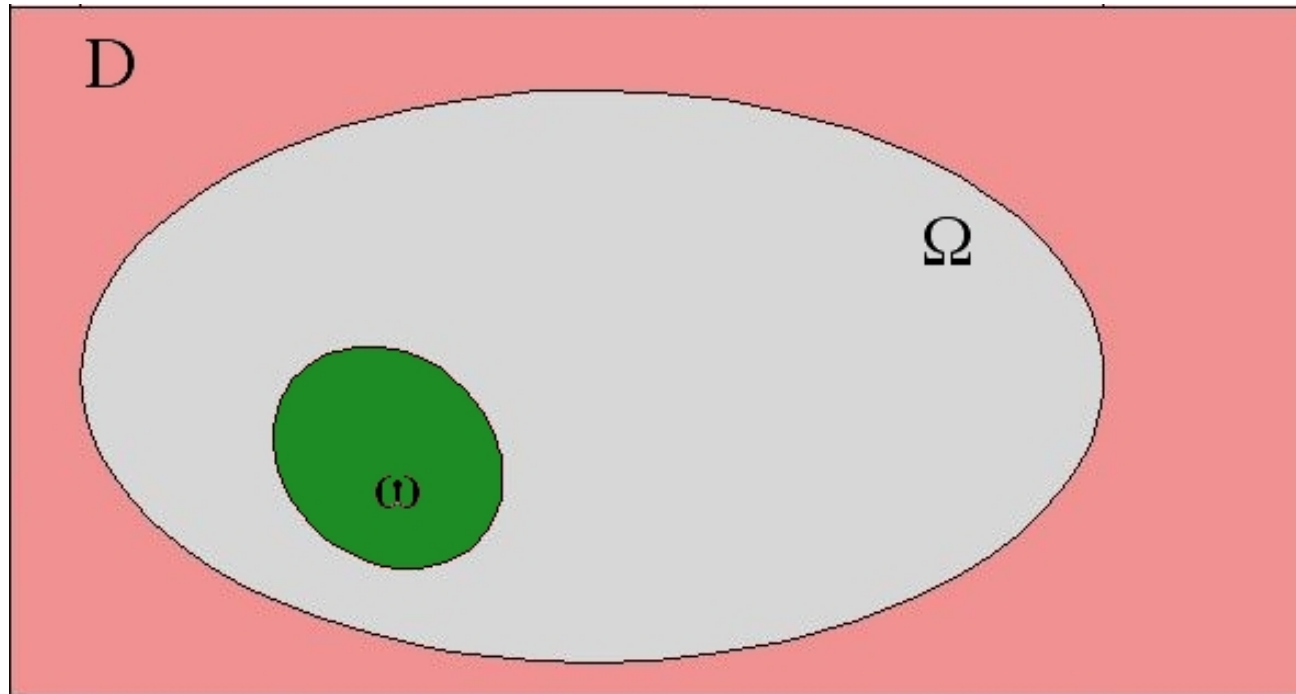
- Desarrollo de hardware específico.
- Desarrollo de nuevos entornos o laboratorios de simulación donde la interacción entre hombre y maquina sea mas rápida y fácil. Por ejemplo mediante la combinación de herramientas mas clásicas de CFD y optimización con técnicas de la geometría y análisis de imágenes.

A MATHEMATICAL EXAMPLE

FINITE ELEMENT APPROXIMATION OF OPTIMAL SHAPE DESIGN PROBLEMS

D. Chenais & E. Z., Finite Element Approximation of 2D Elliptic Optimal Design, JMPA, 85 (2006), 225-249.

M. Cea & E. Z., Finite Element Approximation of 2D Parabolic Optimal Design Problems, Numerical Mathematics and And Advanced Applications, ENUMATH 2005, A. Bermúdez de Castro et al., eds, Springer, 2006, pp. 151-176.



To choose the optimal domain Ω within the class of domains, embedded in the pavé D and containing the subdomain ω .

The number of applications is huge:

- * Design of flexible structures.
- * Location of pollutants.
- * Optical fibers, wave guides.
- * Medicine, Biology,...

OPTIMAL DESIGN

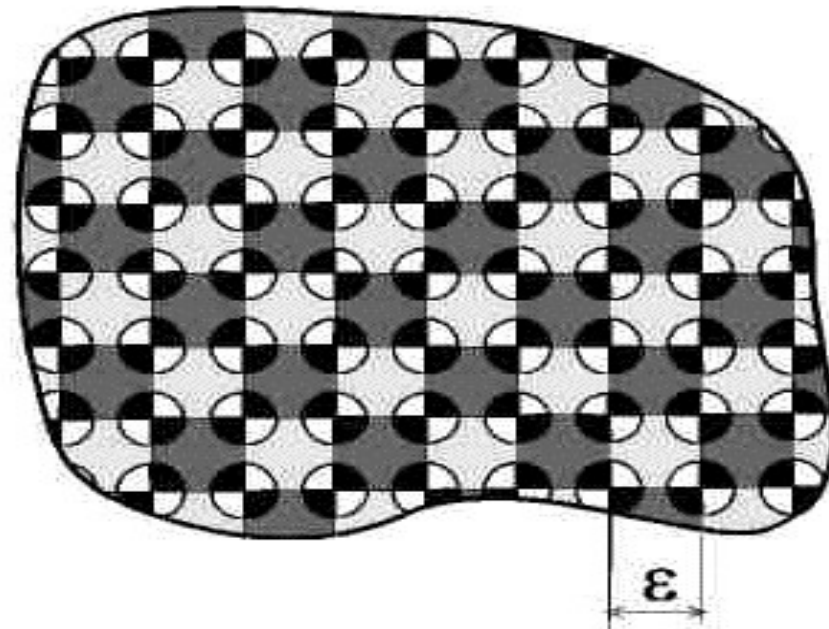
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OPTIMIZATION PROCESS INVOLVING GEOMETRIES AND SHAPES.

Elliptic optimal design. Control = Shape of the domain. State equation = Dirichlet Laplacian.

Dimension $n = 2$, Šverák: There exists an optimal domain in the class of all open subsets of a given bounded open set, whose complements have a uniformly bounded number of connected components.

Key point: compactness of this class of domains with respect to the complementary-Hausdorff topology and the continuous dependence of the solutions of the Dirichlet Laplacian in H^1 with respect to it.



It is well known that, when the number of holes is unlimited, homogenization phenomena arise and the minimum is not achieved. Cioranescu-Murat: $-\Delta \rightarrow -\Delta + \mu$.

COMMON COMPUTATIONAL/NUMERICAL PRACTICE:

- * **Continuous optimal design** → discrete finite-element version.
- * **Compute the discrete optimal shape** (discrete optimization or shape and topological derivatives, level set methods,...)

The choice of one method or another depends very much on the expertise and computational capacities.

THE PROBLEM:

Do these methods converge? **YES!**

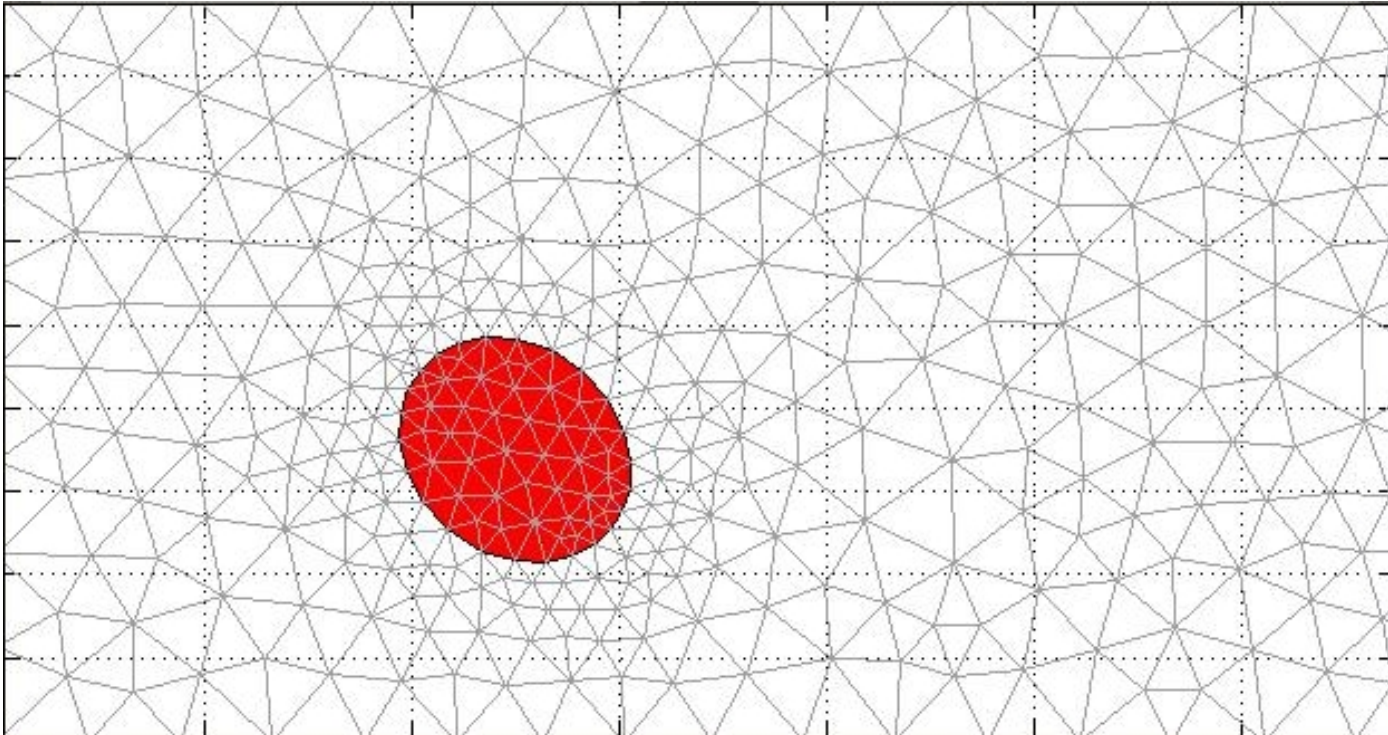
Key point : finite-element approximations of the solution of the Dirichlet laplacian converge in H^1 whenever the polygonal domains converge in the sense of H^c -topology.

This provides a rigorous justification to the most common engineering procedures in numerical simulation of optimal design problems.

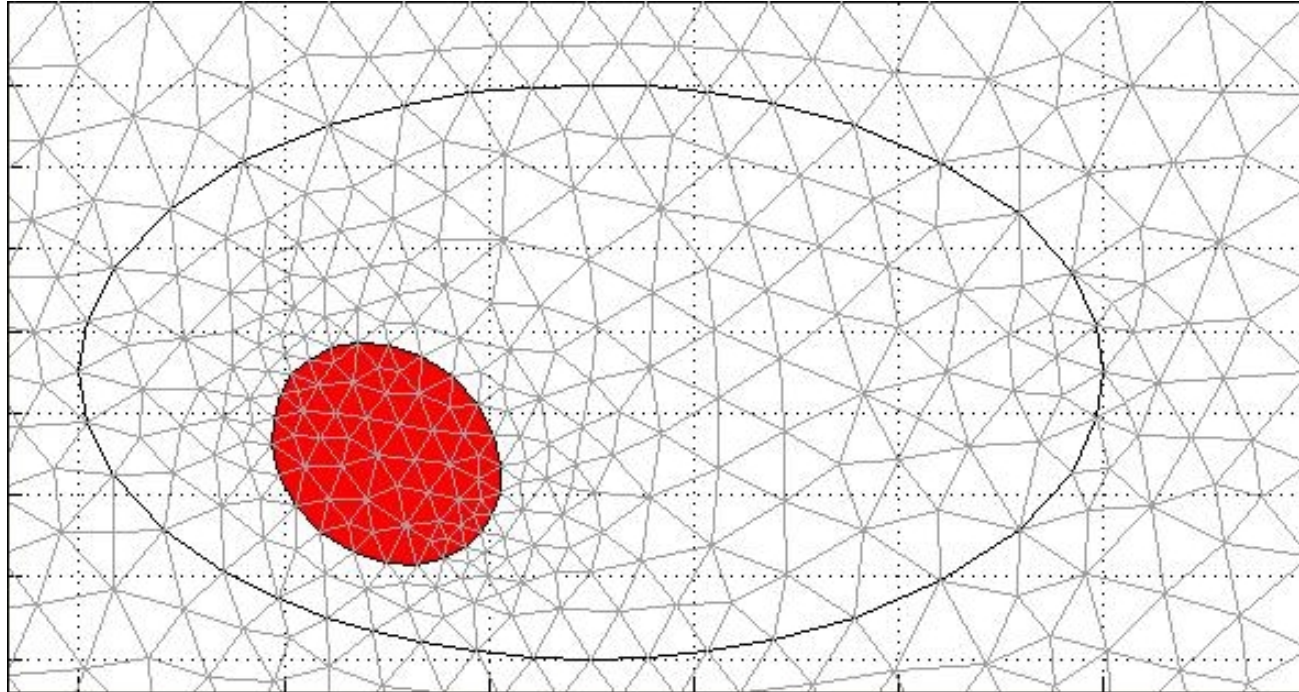
OPTIMAL SHAPE DESIGN+NUMERICS

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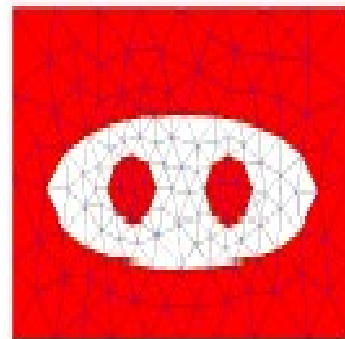
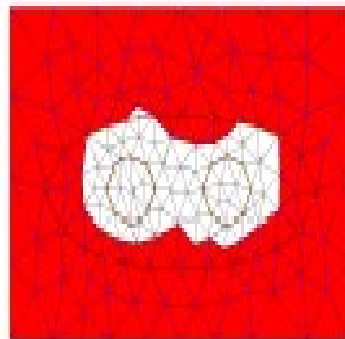
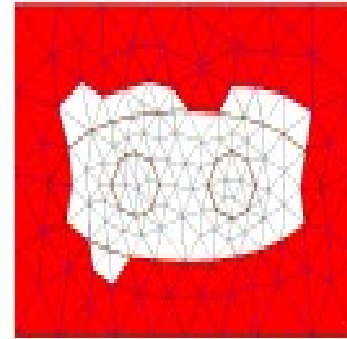
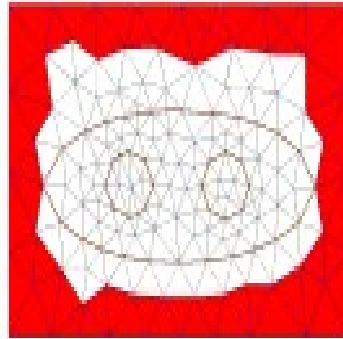
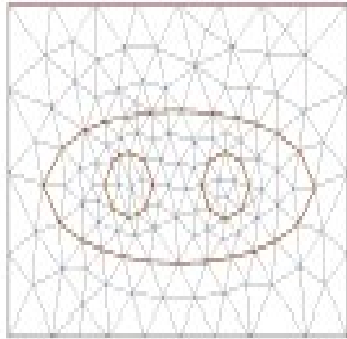
NUMERICS+OPTIMAL SHAPE DESIGN



The triangulation of the pavé and the fixed subdomain (constraint) from which all admissible discrete domains have to be built.



The class of admissible domains for the discrete problem. This time the admissible domains need to be unions of triangles from the discrete mesh.



CONCLUSION:

- * Control and Optimal Design Problems are ubiquitous:
- * There is an important need of using deep tools of mathematics: PDE, Numerical Analysis and Simulations, etc.
- * There are challenging and difficult open problems requiring important efforts. In particular melting continuous and discrete approaches should be a priority.