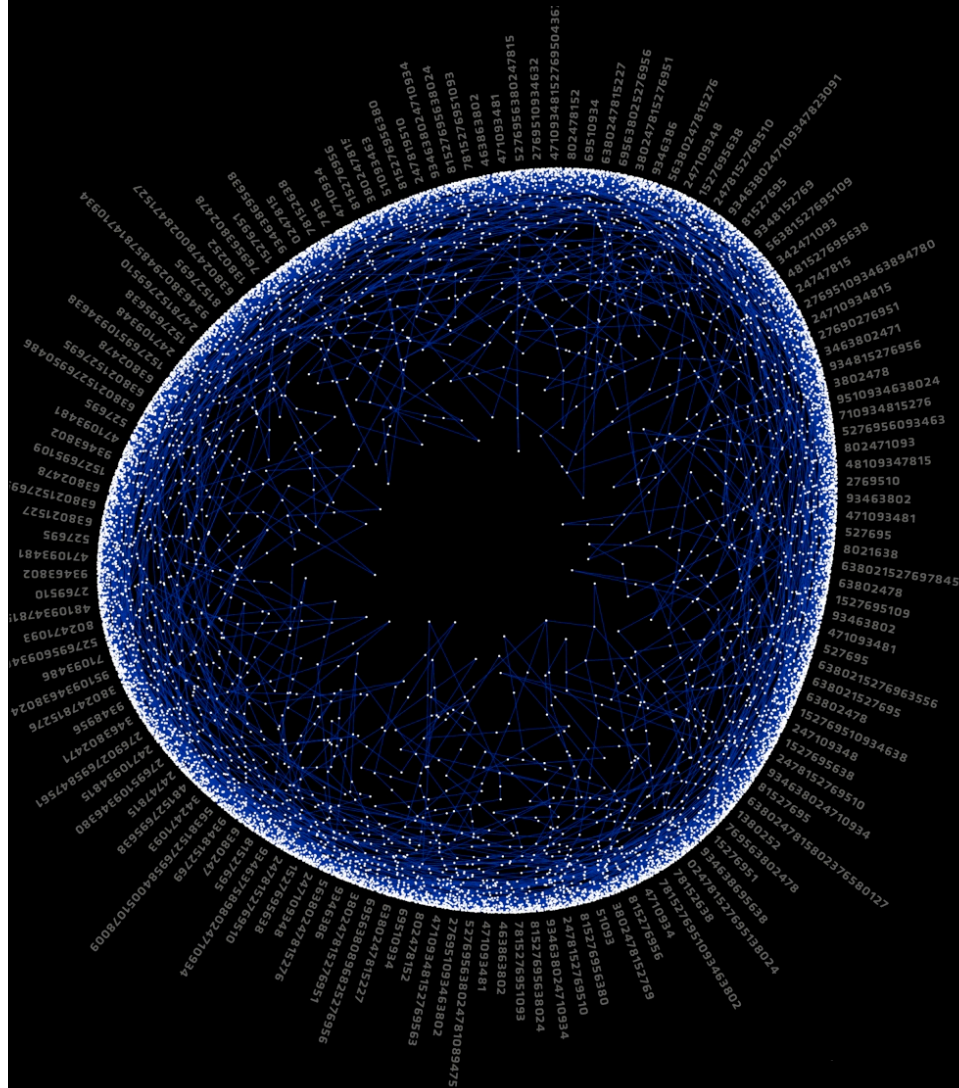


MATEMOZIOA

Matematikaren hitzordua | Cita con las Matemáticas



Mathematical modeling: from the Galileo legacy to the environment, medicine and technology

Alfio Quarteroni

EPFL, Lausanne (Switzerland)

MATHICSE Mathematics Institute of Computational
Science and Engineering

POLITECNICO di MILANO (Italy)

MOX Modellistica e Calcolo Scientifico

21/02/2012 | 19:00

BIZKAIA ARETOA | PARANINFO UPV/EHU



**POLITECNICO
DI MILANO**



CMCS



EPFL
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Medicine,
Technology,
Environment

Why Math Matters?

Two Instances from our daily life..

- Fast internet search thanks to **efficient matrix computation**
- Image transmission by **fast and accurate compression algorithms**



Searching on the web

230.000.000

Two hundred thirty millions

Websites

1.000.000.000.000

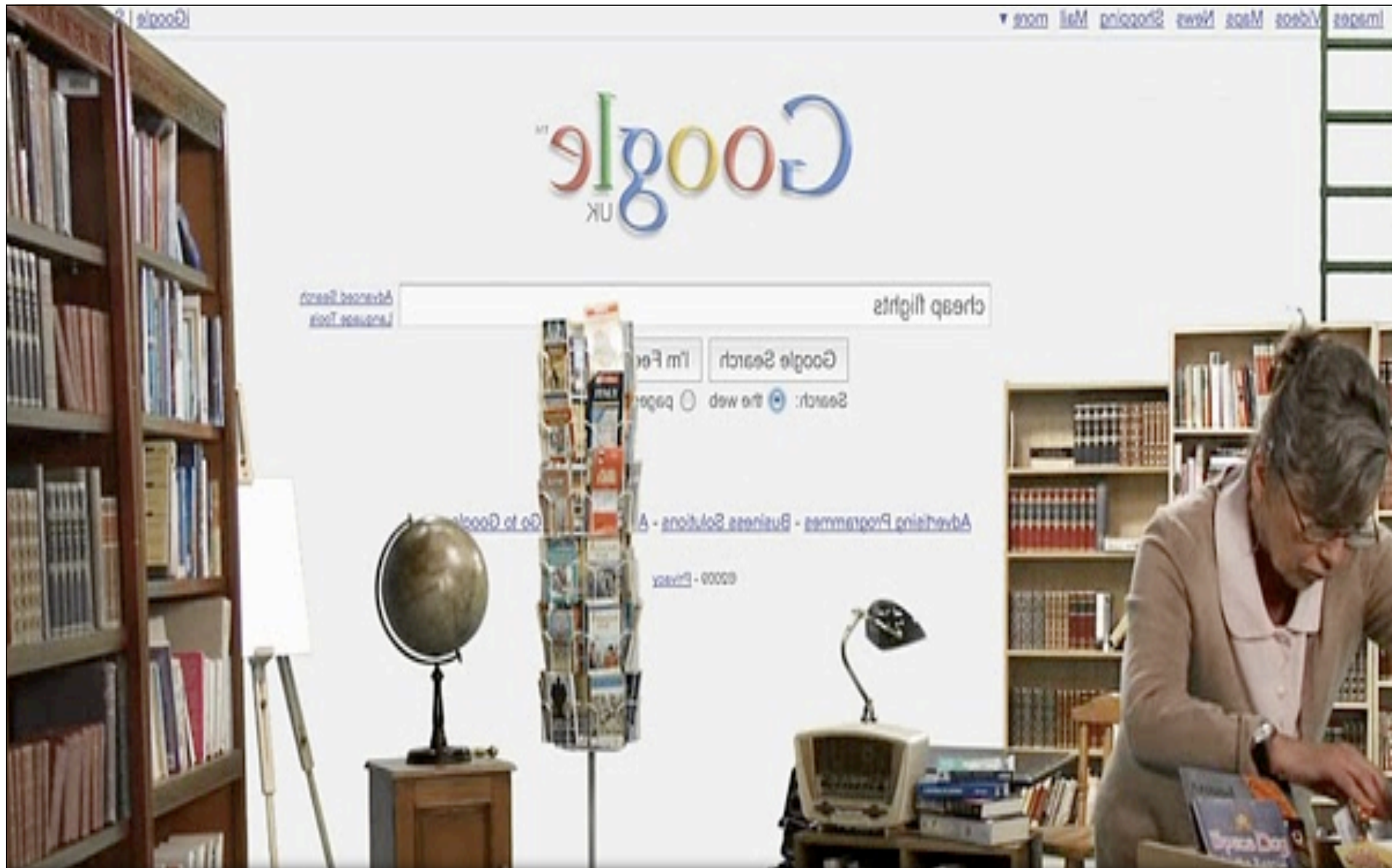
A thousand billions

Webpages

1.500.000.000

A billion and a half

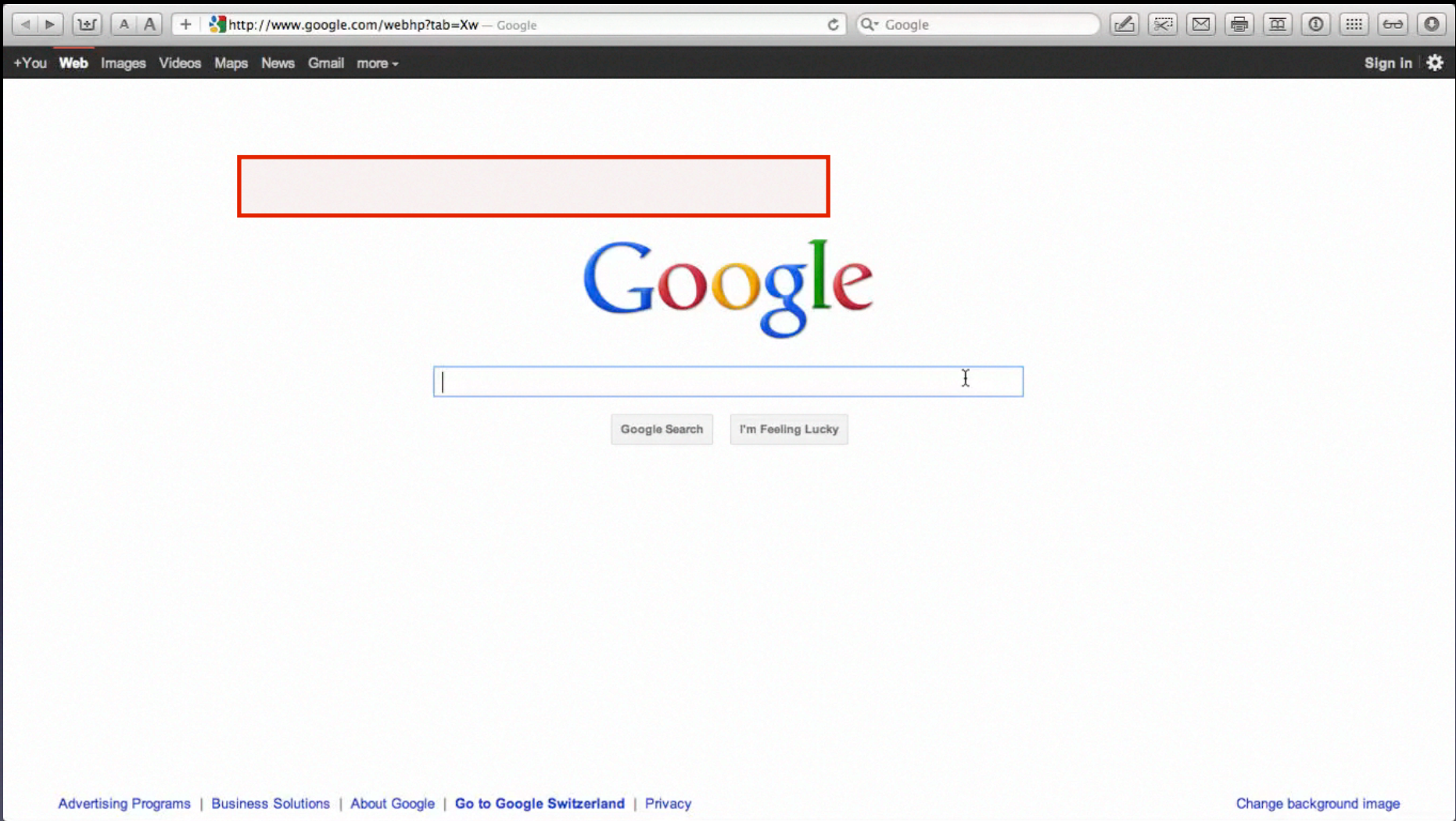
Users



How to find out what we're looking for?



Search Engines



Googling for Isaac Newton

Among

1.000.000.000.000

Thousand Billion

Webpages

Google selects

10.000.000

Ten Million

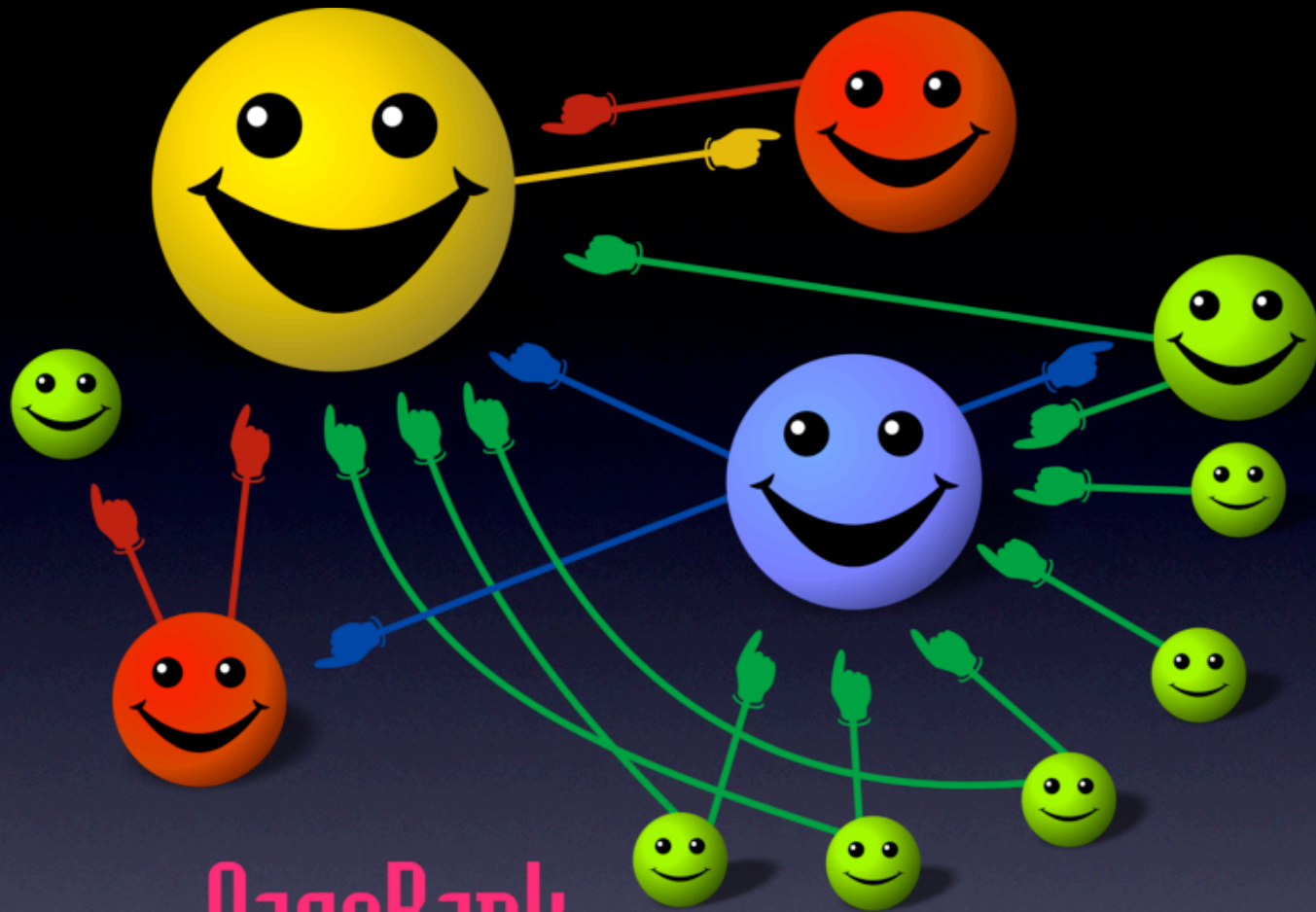
for us

in just

14

Fourteen

centiseconds



PageRank

$$PR(A) = \frac{1-d}{N} + d \left(\frac{PR(B)}{L(B)} + \frac{PR(C)}{L(C)} + \frac{PR(D)}{L(D)} + \dots \right).$$

Google

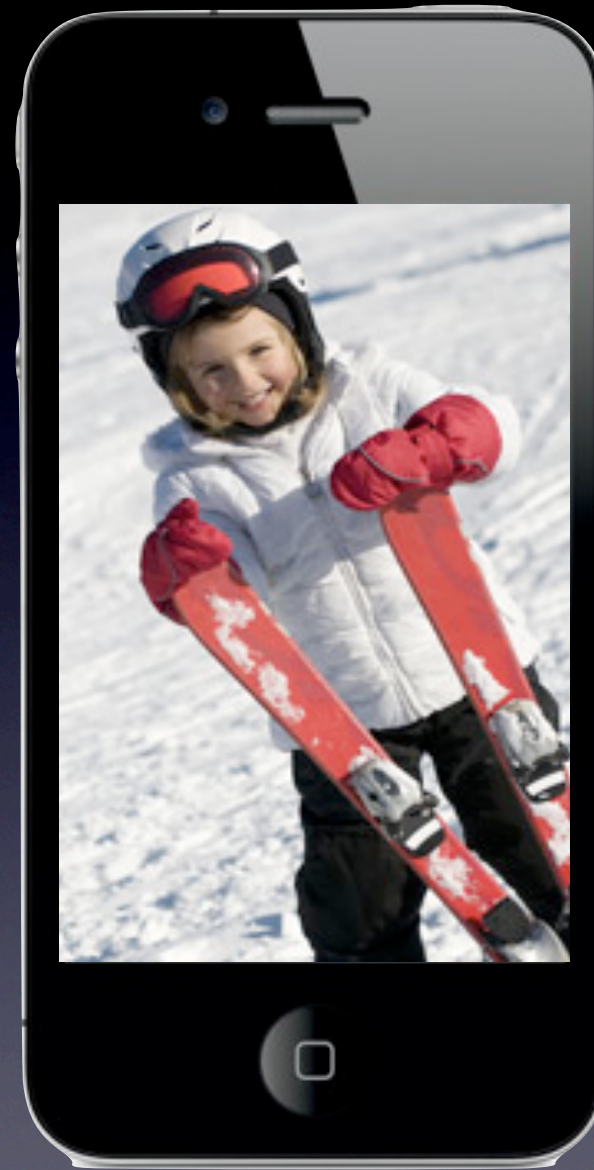
is so fast thanks to

Mathematics!

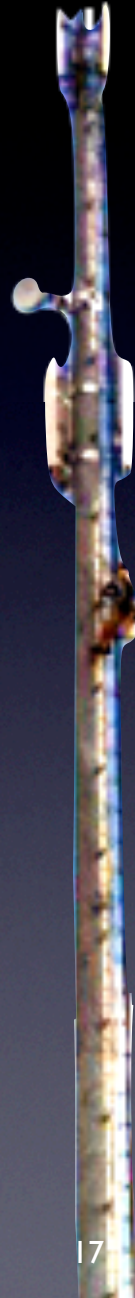


Sending pictures
from our mobile phone

What you see..

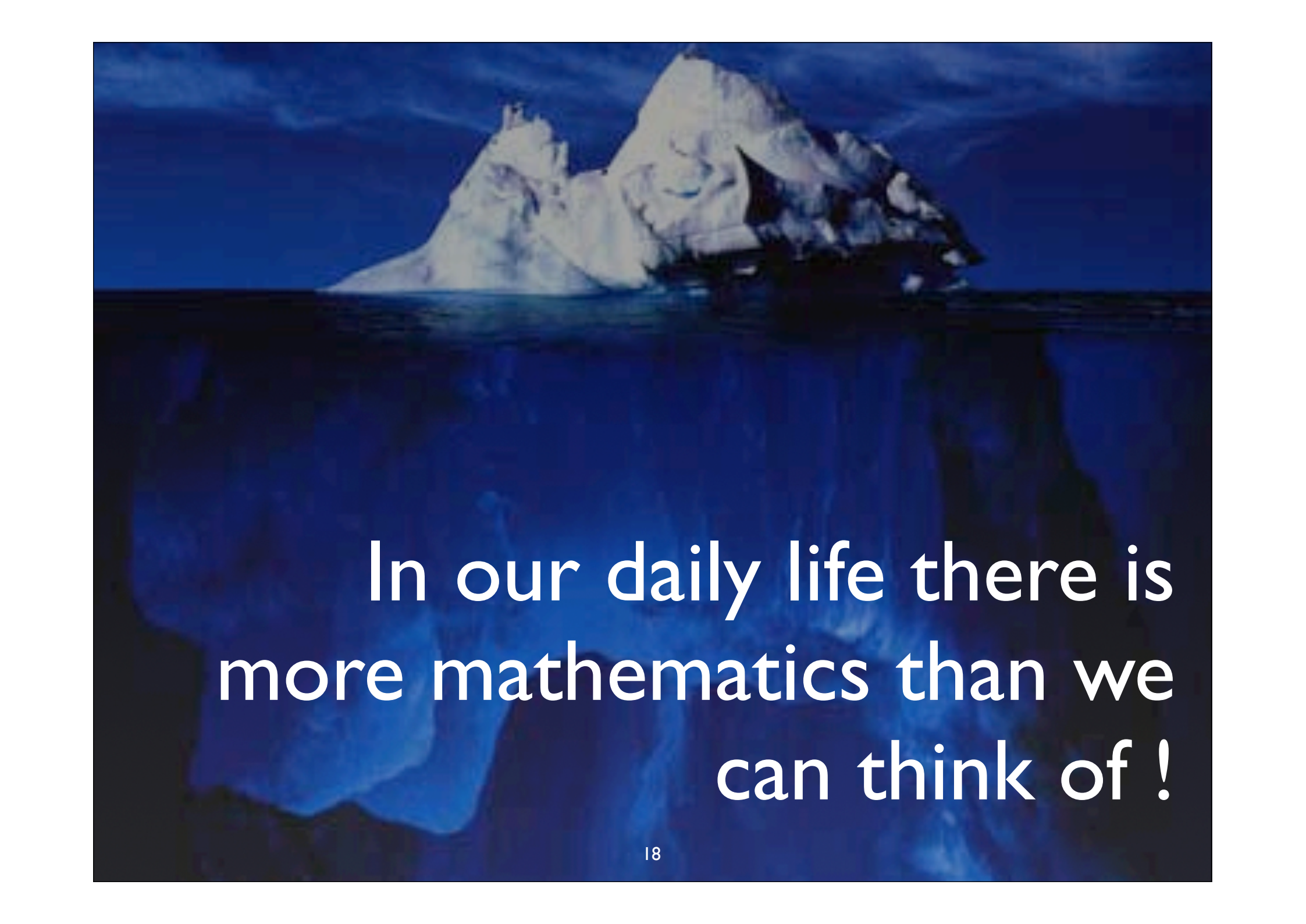


... and what does really happen



FWT
Fast Wavelet
Transform

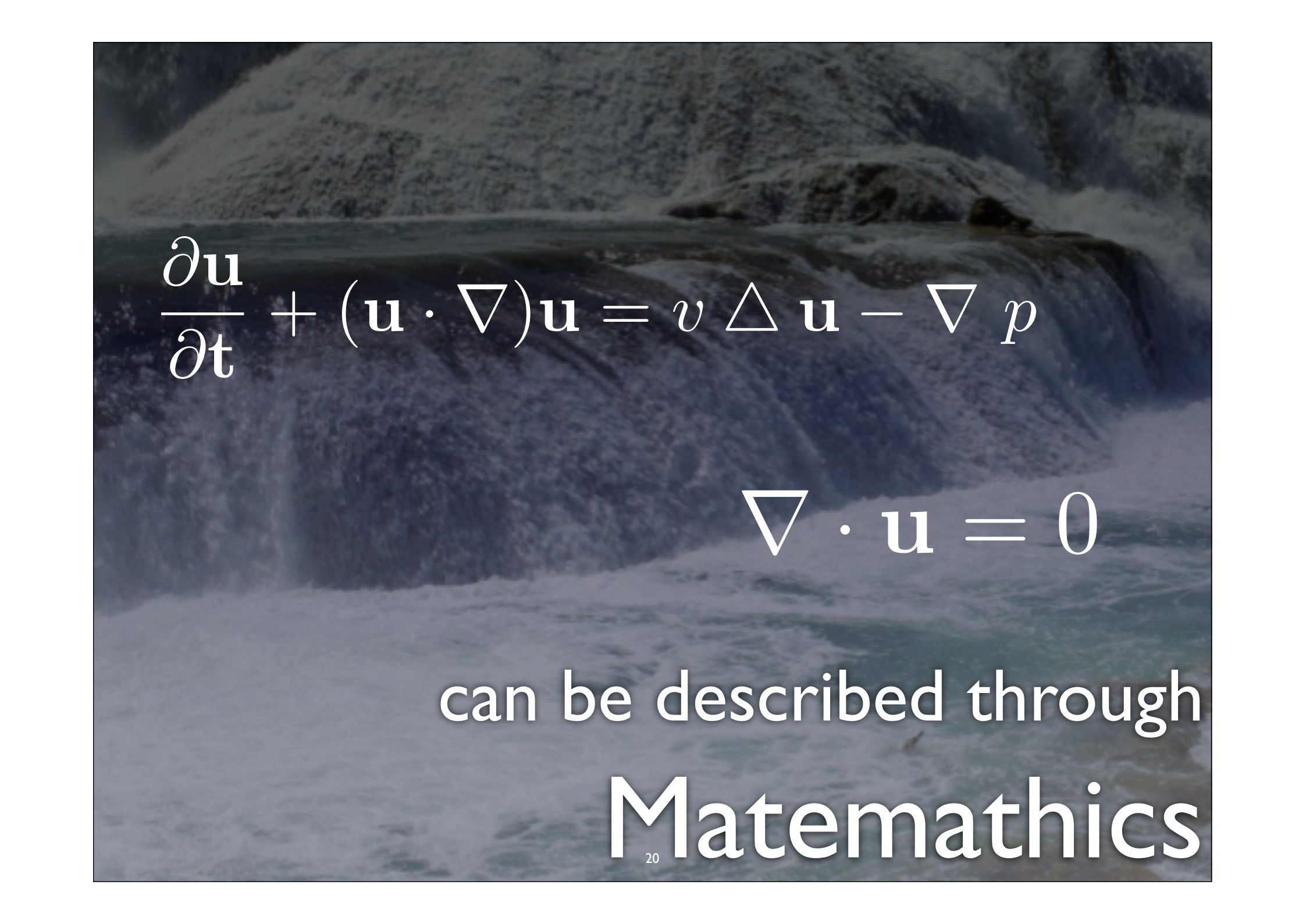
Inverse FWT

A photograph of an iceberg floating in the ocean. The visible tip of the iceberg is small and jagged, while the vast, dark, and submerged part of the iceberg is much larger, illustrating the concept of hidden complexity. The sky is a deep blue, and the water is a darker blue.

In our daily life there is
more mathematics than we
can think of !



water


$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = \nu \Delta \mathbf{u} - \nabla p$$

$$\nabla \cdot \mathbf{u} = 0$$

can be described through

Mathematics



sea waves



are Mathematics

$$u_t + \left(\frac{u^2}{2} \right)_x + \varepsilon^2 u_{xxx} = 0$$

predator-prey competition



A photograph of a lion lying in a savanna landscape with tall, dry grass. The lion is the central focus, looking slightly to the left. Overlaid on the image are two mathematical equations in white text. The first equation is $u_t = d_1 \Delta u + a u - b u v - f u^2$ and the second is $v_t = d_2 \Delta v - d v + c u v - e v^2$.
$$u_t = d_1 \Delta u + a u - b u v - f u^2$$

$$v_t = d_2 \Delta v - d v + c u v - e v^2$$

is Mathematics



the stripes of
Zebras

the appearance of
Butterflyfish





all that is Mathematics

$$u_t = \Delta u + \gamma f(u, v)$$
$$v_t = d\Delta v + \gamma g(u, v)$$

Key is

MATHEMATICAL MODELING



Math Models describe
World through
Equations



The real world



THE
MODEL

30

Galileo Galilei, 1564 - 1642



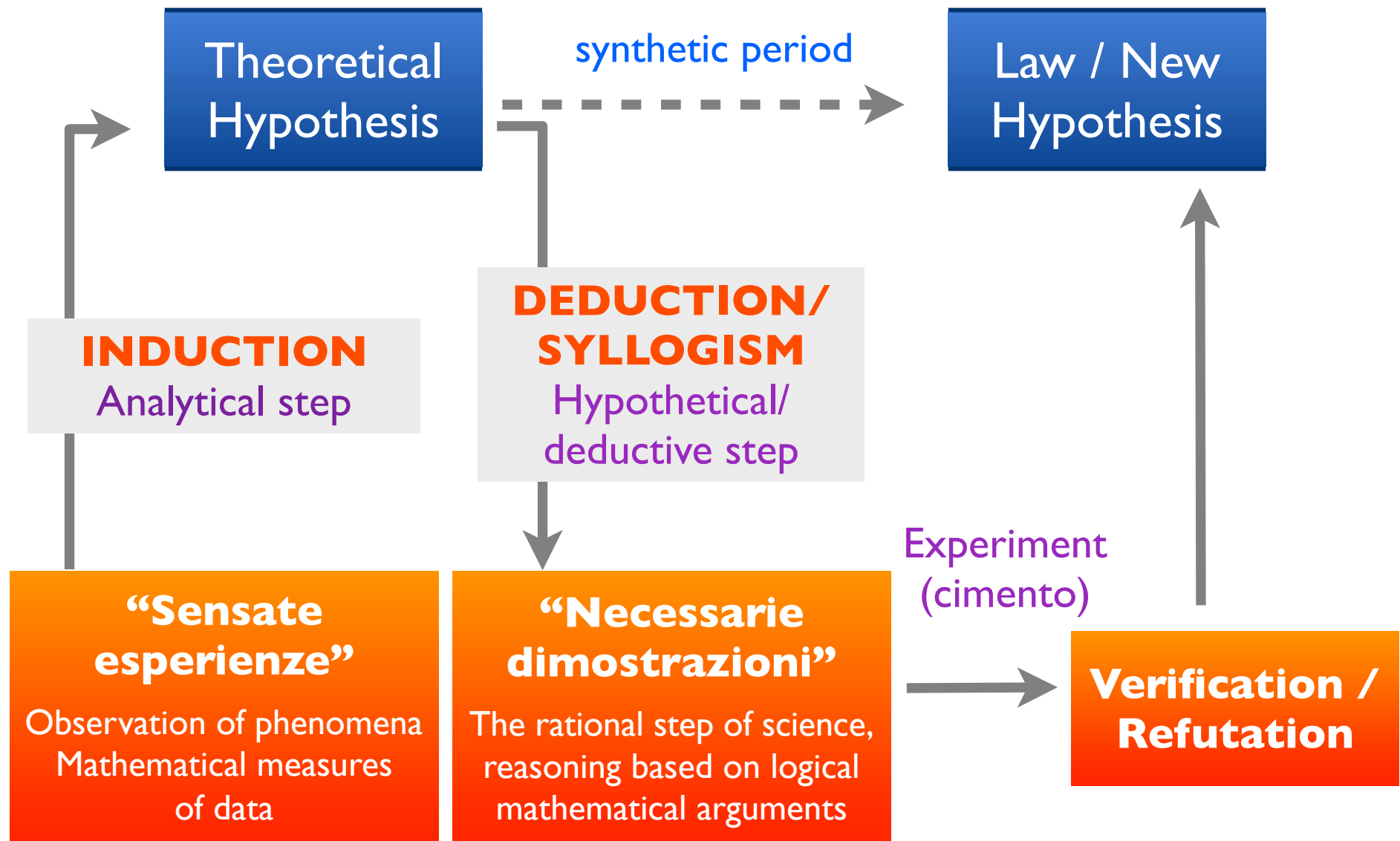
The Book of Nature (Il Saggiatore, 1623)

In 1623 Galileo crafted a famous metaphor (which he used to defend the work of scientists from religious authorities):

*Nature - he wrote - is a book written in
"the language of mathematics".
If we cannot understand that language,
we will be doomed to wander about
as if "in a dark labyrinth".*

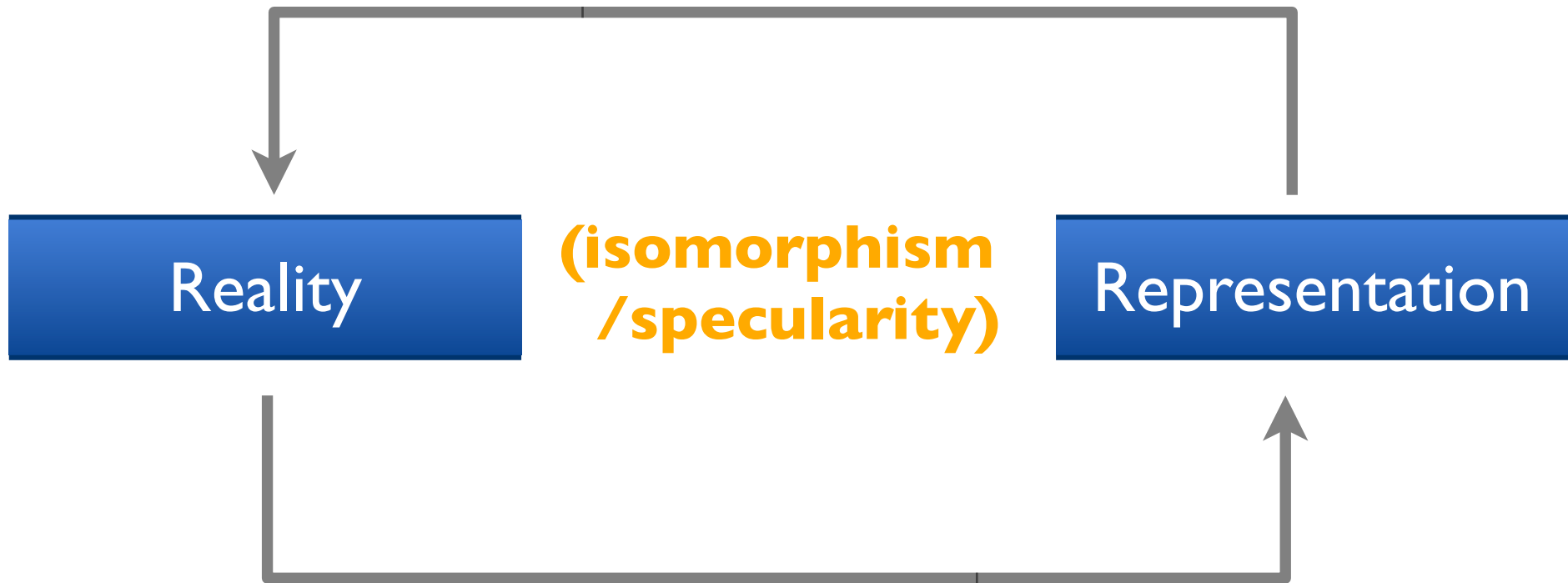
It captures our sense that nature's truths are somehow imposed on us – that they are already imprinted in the world – and **underlines the key role played by mathematics in expressing those truths.**

Towards Mathematical Modeling: the Galilean Method



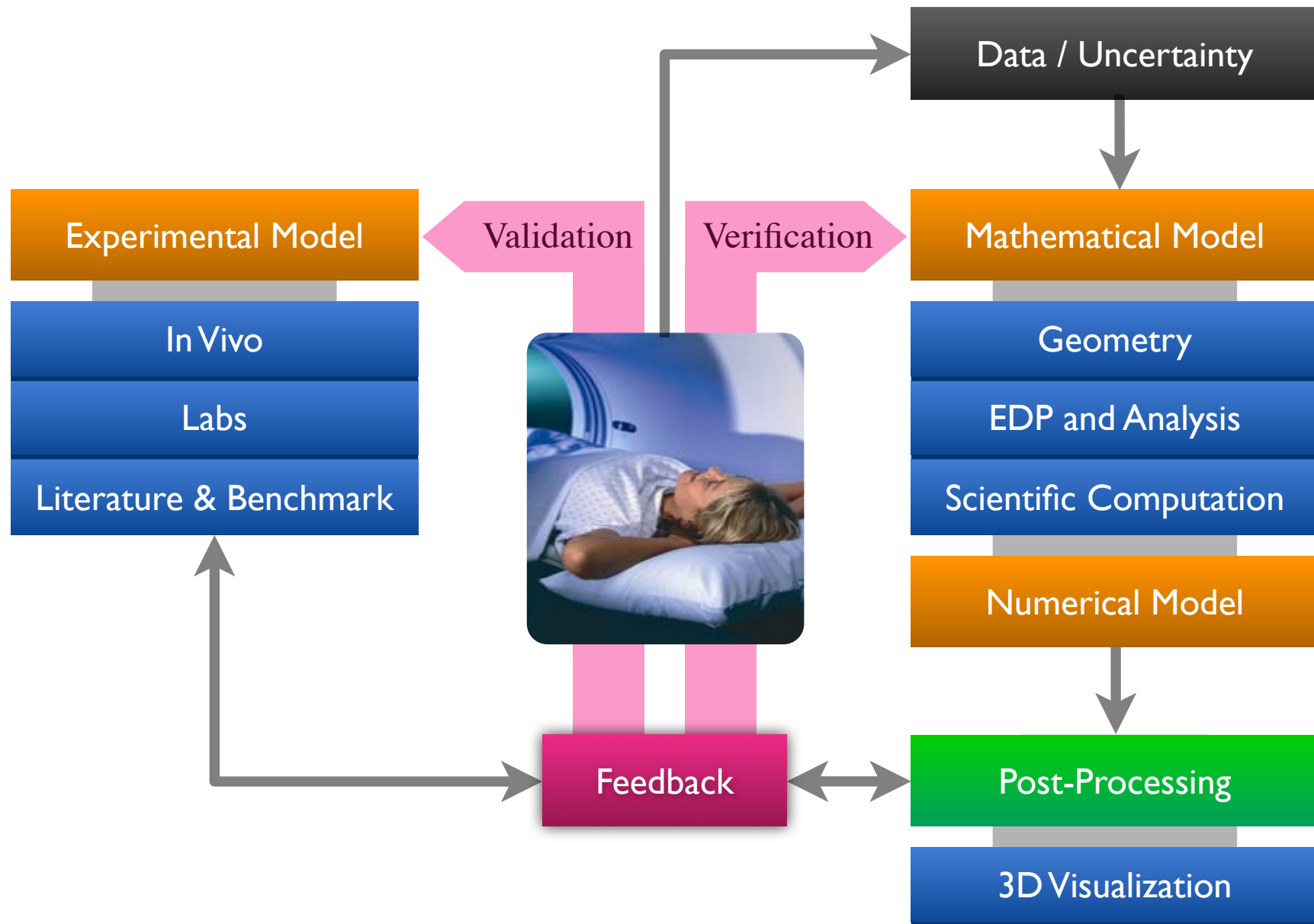
The Paradigm

SCIENCE



**REALISTIC
META-PHYSICS**

Mathematical Modeling and Simulation: an Outlook

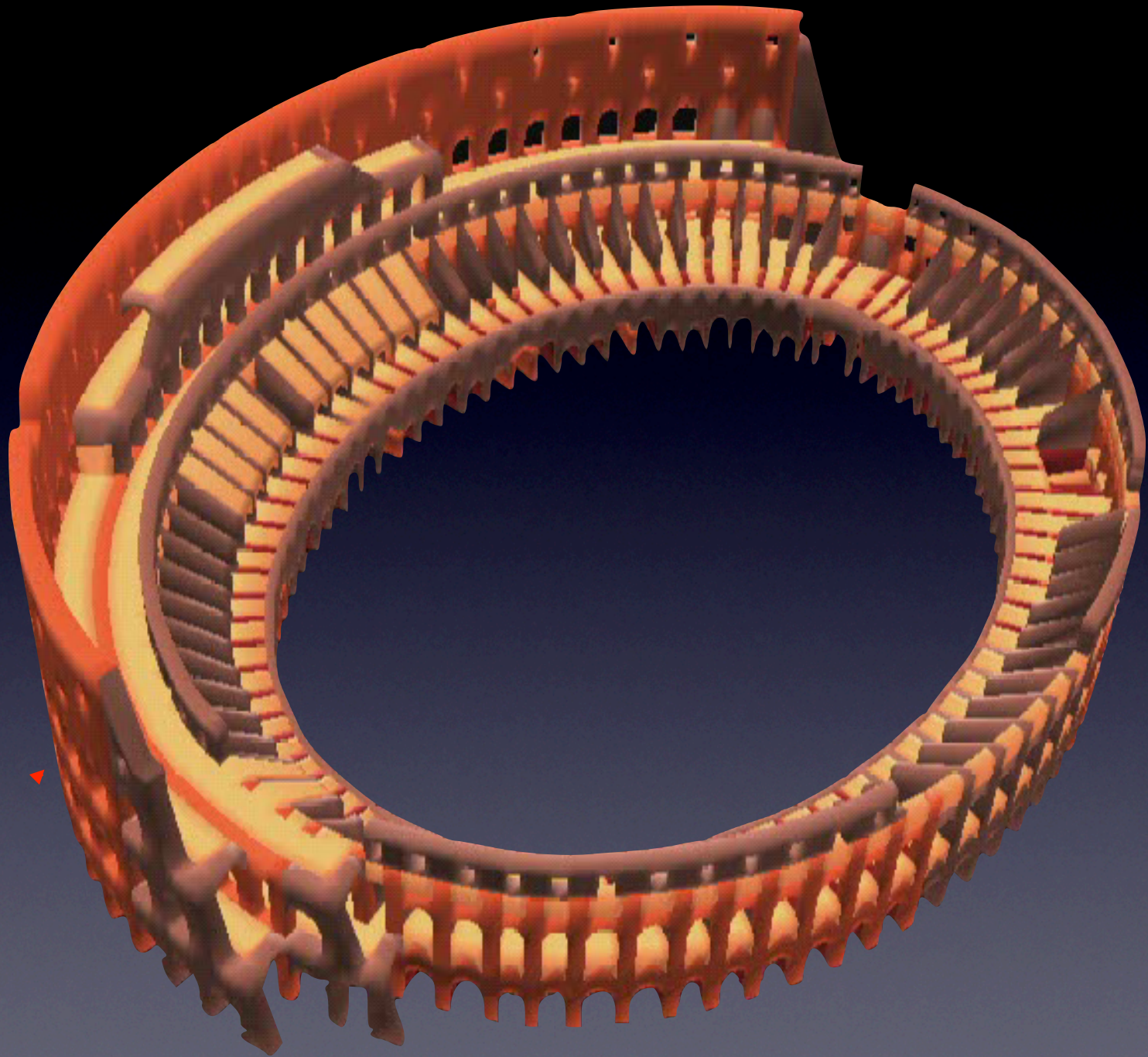


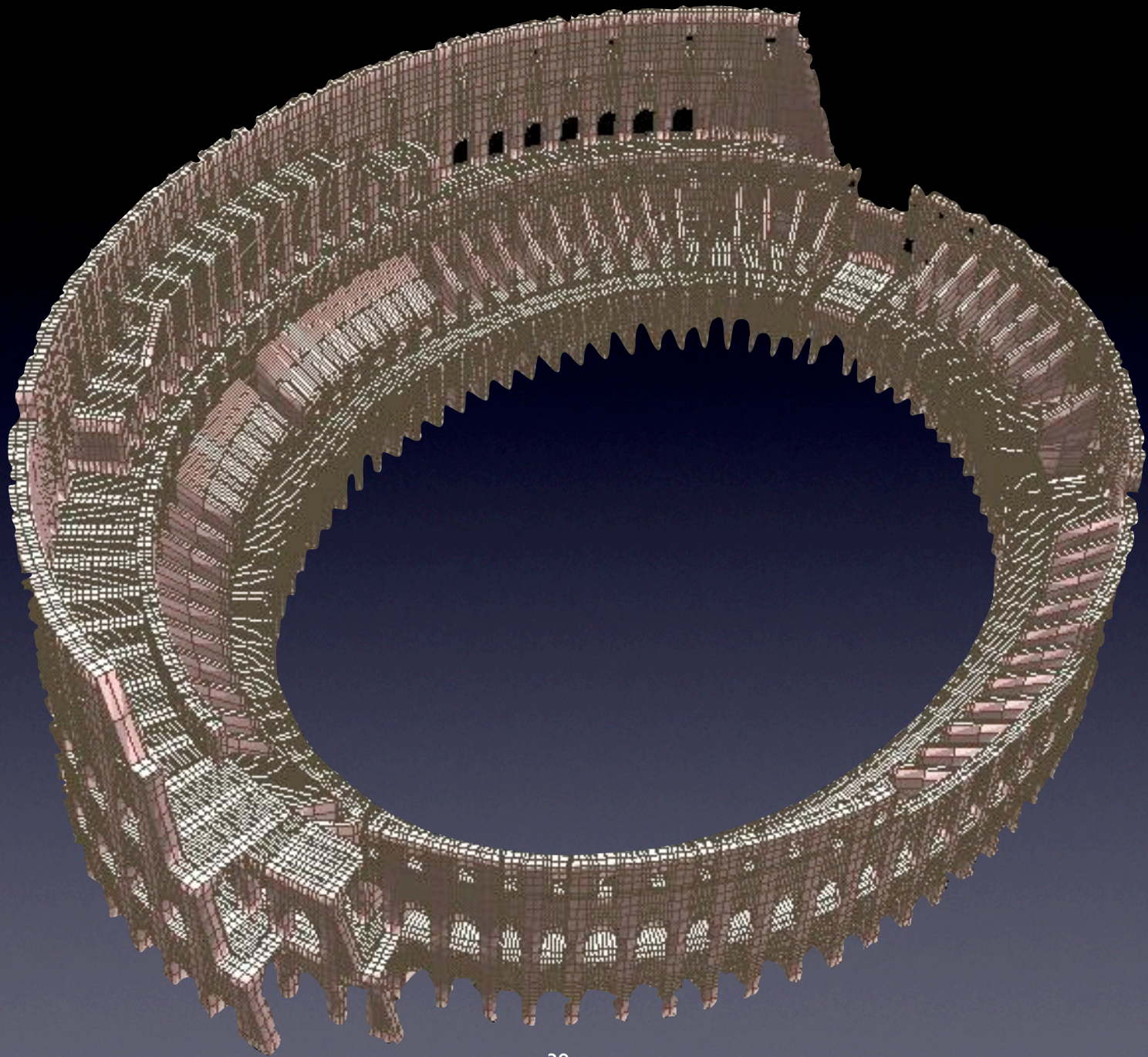
Mathematics
is our
Immaterial Lab

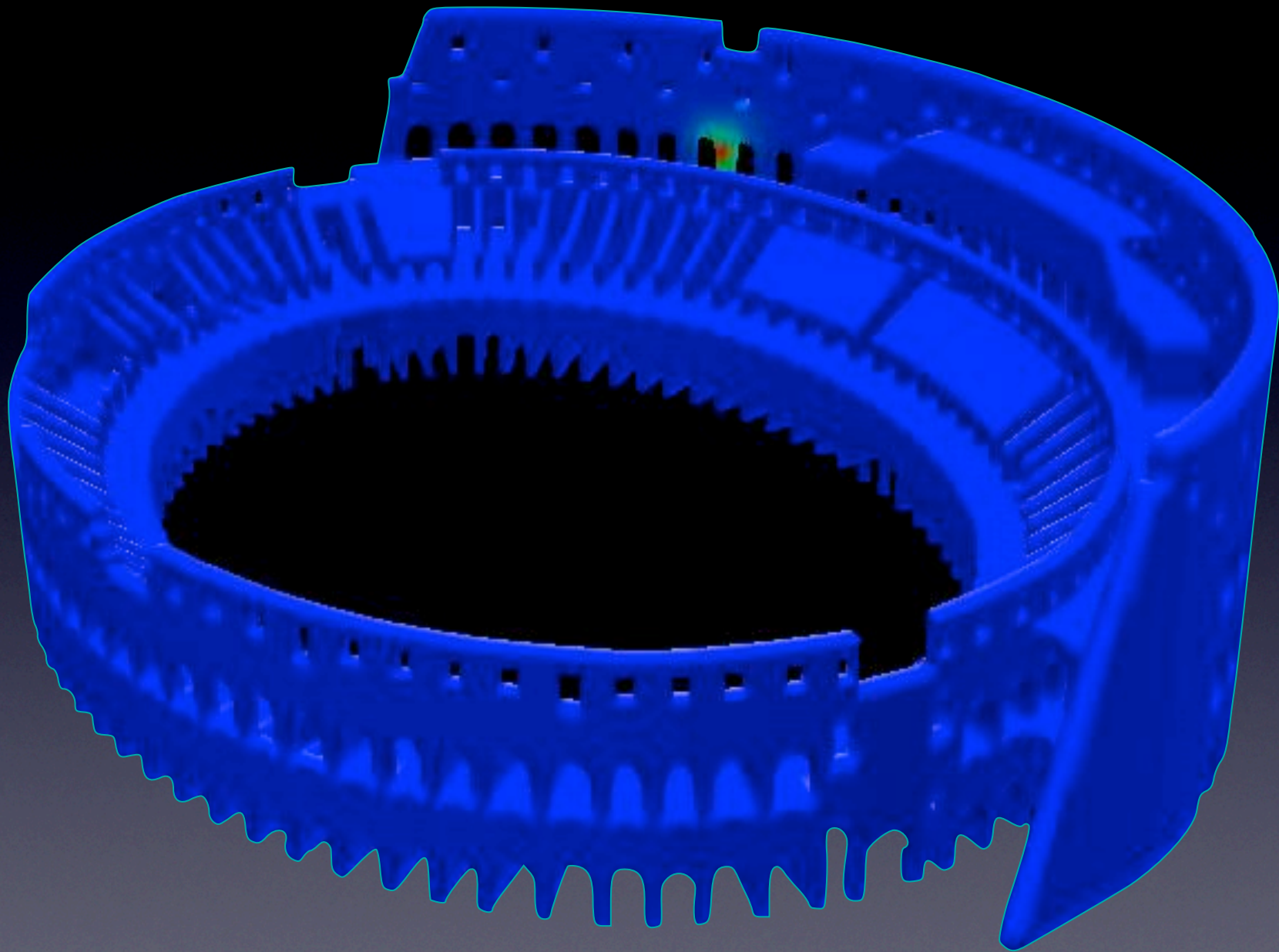


Math is non-destructive

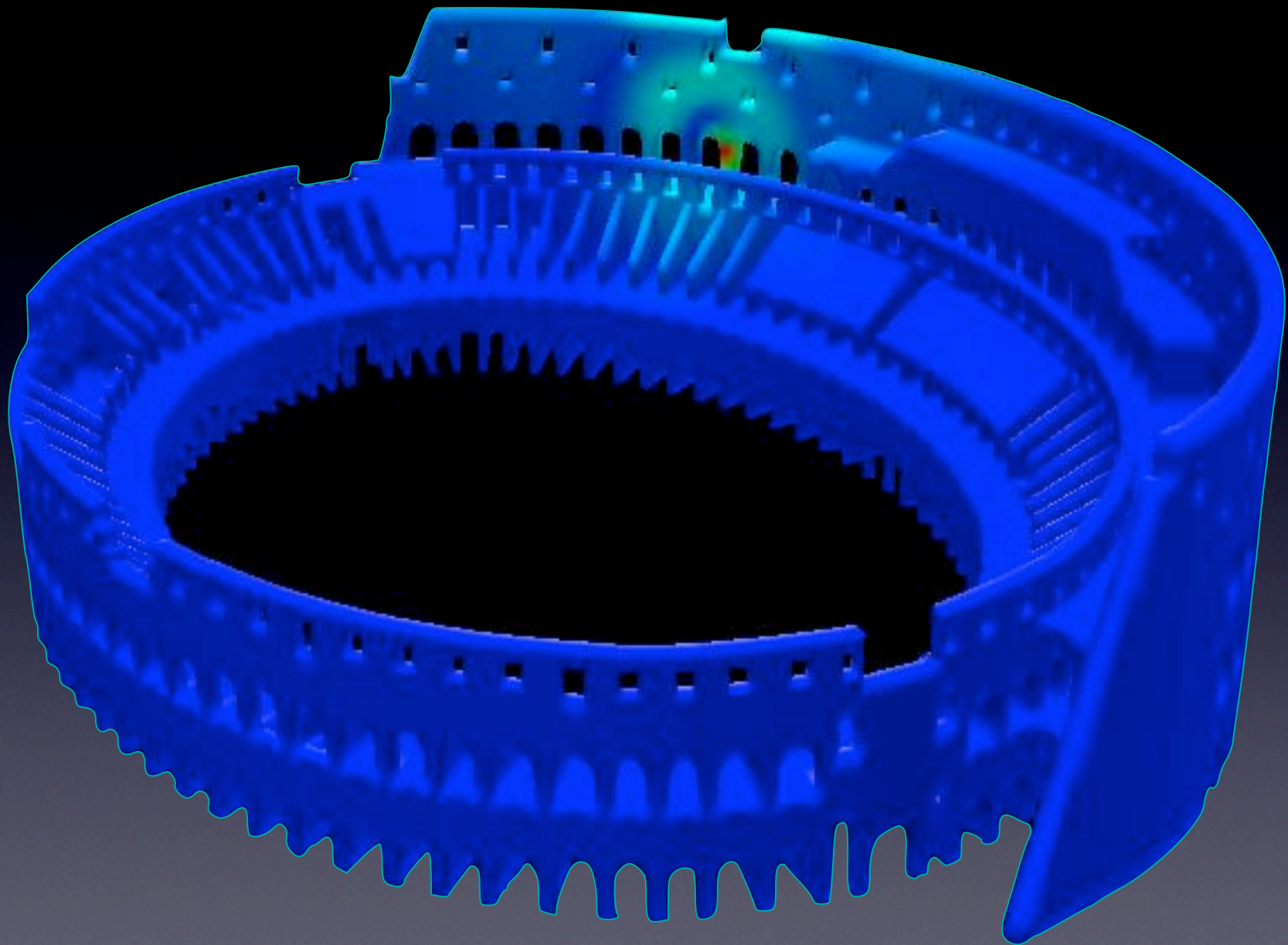
Can we predict the
impact of
earthquakes?



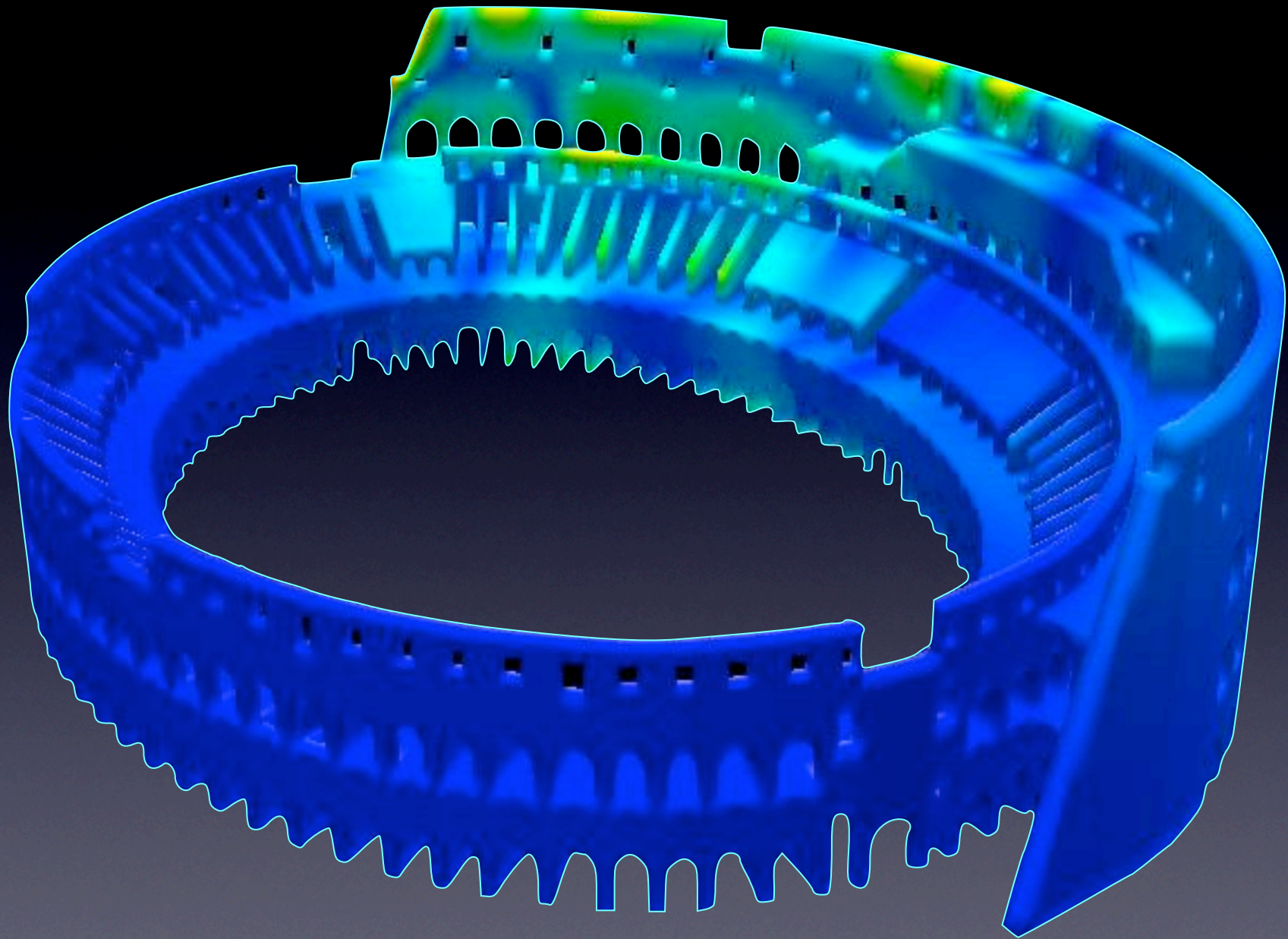




Maggio-Massidda-Fotia @ CRS4



Maggio-Massidda-Fotia @ CRS4



Maggio-Massidda-Fotia @ CRS4

Seismic Waves Simulation

by high order methods

Seismic response of the
Acquasanta Viaduct
(Liguria - Italy)

Seismic waves propagation in
Christ Church, New Zealand

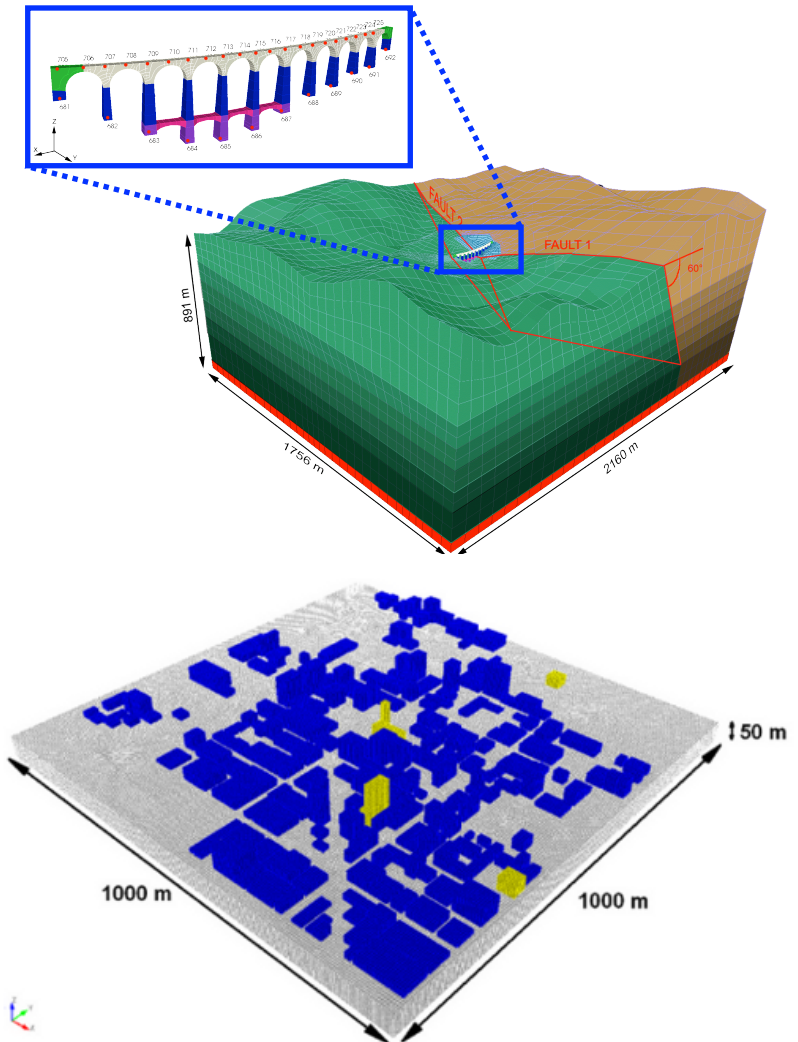
Acknowledgments:

R.Paolucci, Ch.Smerzini, DIS - Politecnico di Milano

M.Stupazzini, Munich RE

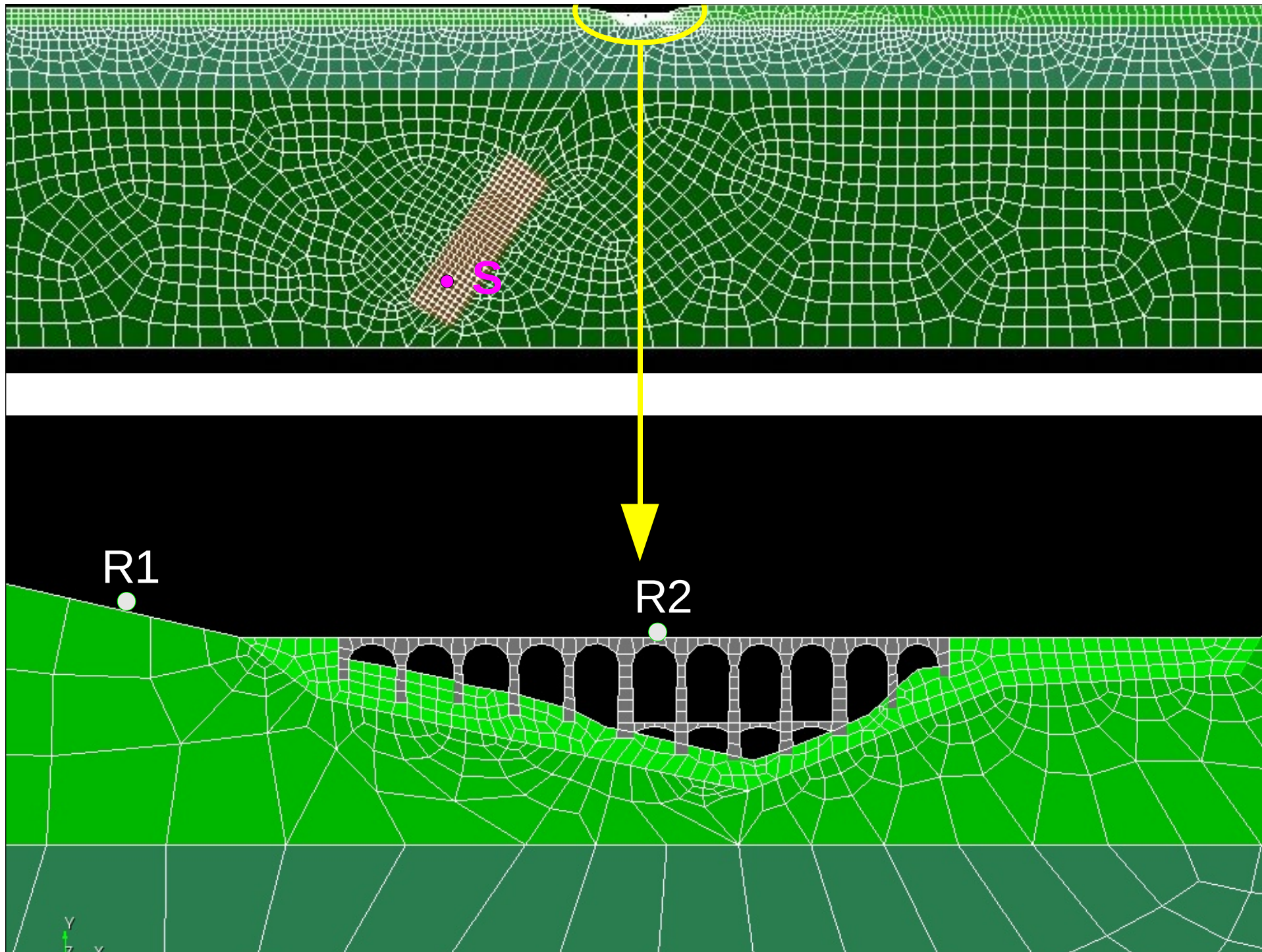
P.Antonietti, I.Mazzieri, MOX - Politecnico di Milano

F.Rapetti, Université de Nice

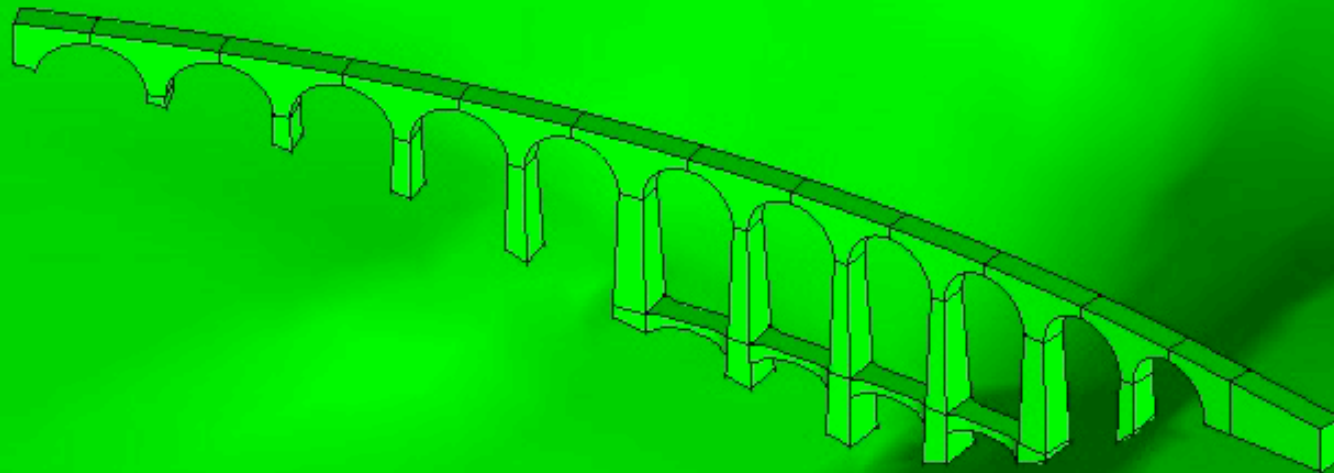


Acquasanta Bridge, Liguria, Italy





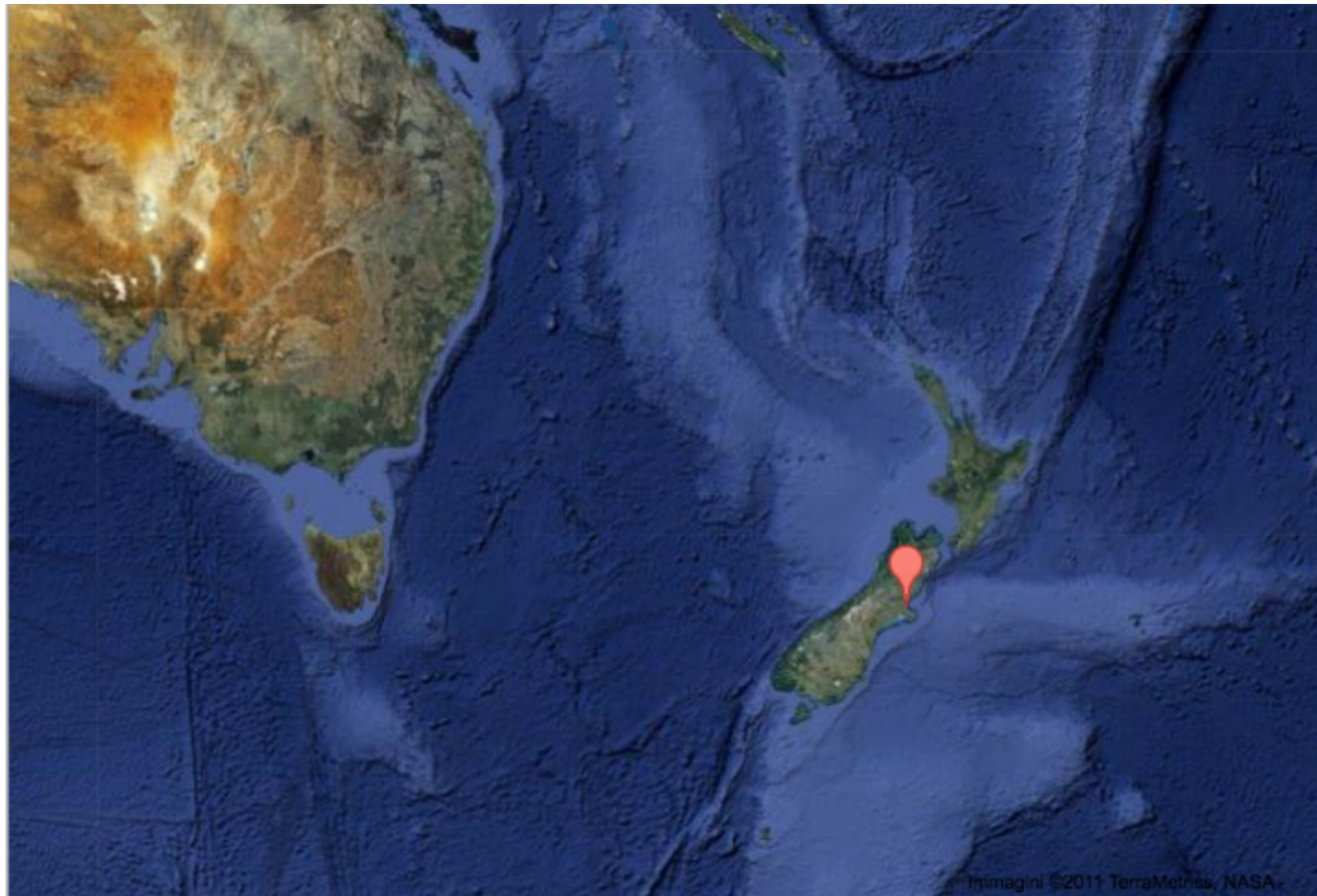
Acquasanta Viaduct, Liguria, Italy



step 0.01
Display Vectors of displ, |displ| factor 5.
Deformation (x5): displ of TIME ANALYSIS, step 0.01.

x

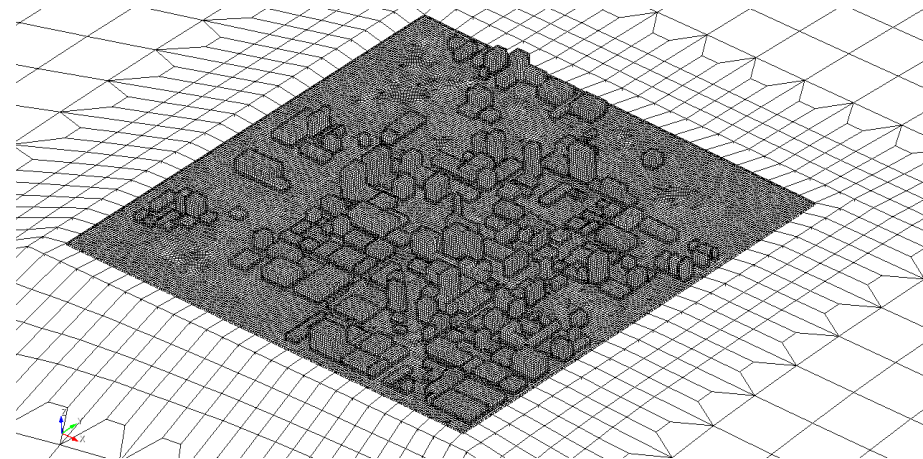
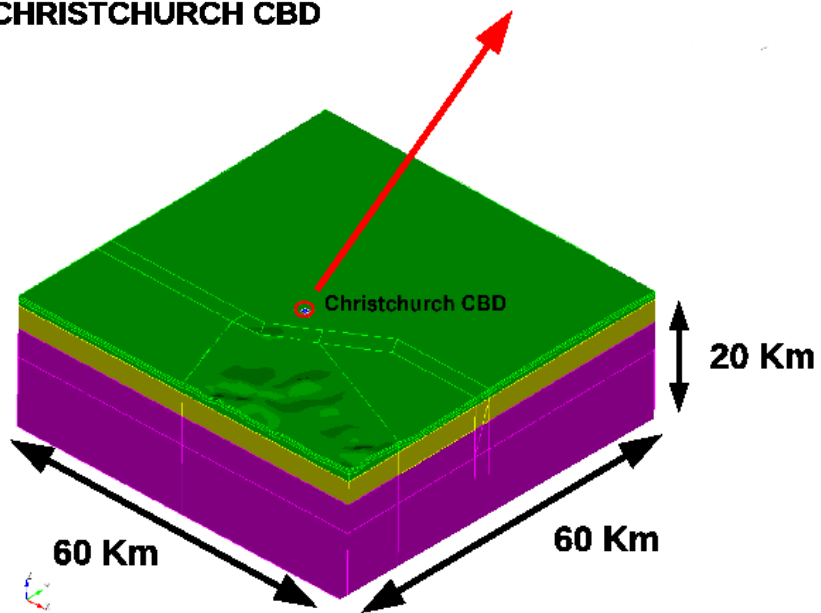
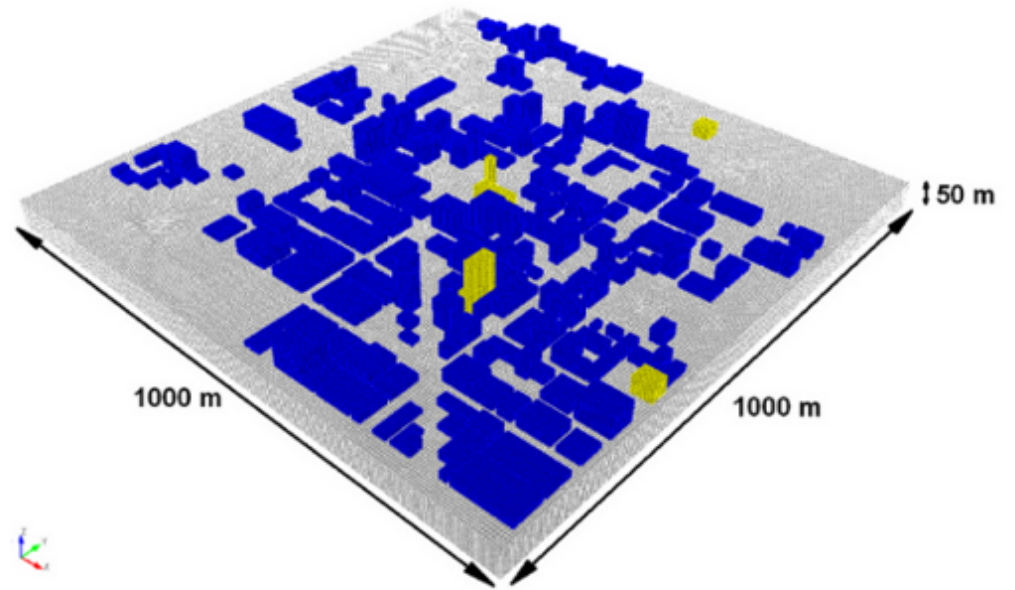
Christ Church NZ (6.2 Richter - 22 February 2011)



Christchurch Central Business District



CHRISTCHURCH CBD



Christ Church NZ (6.2 Richter - 22 February 2011)

Christchurch Business District



Math Models for Human Life

1. From clinical imaging to volume reconstruction

2. Mathematical Modeling

3. Computing

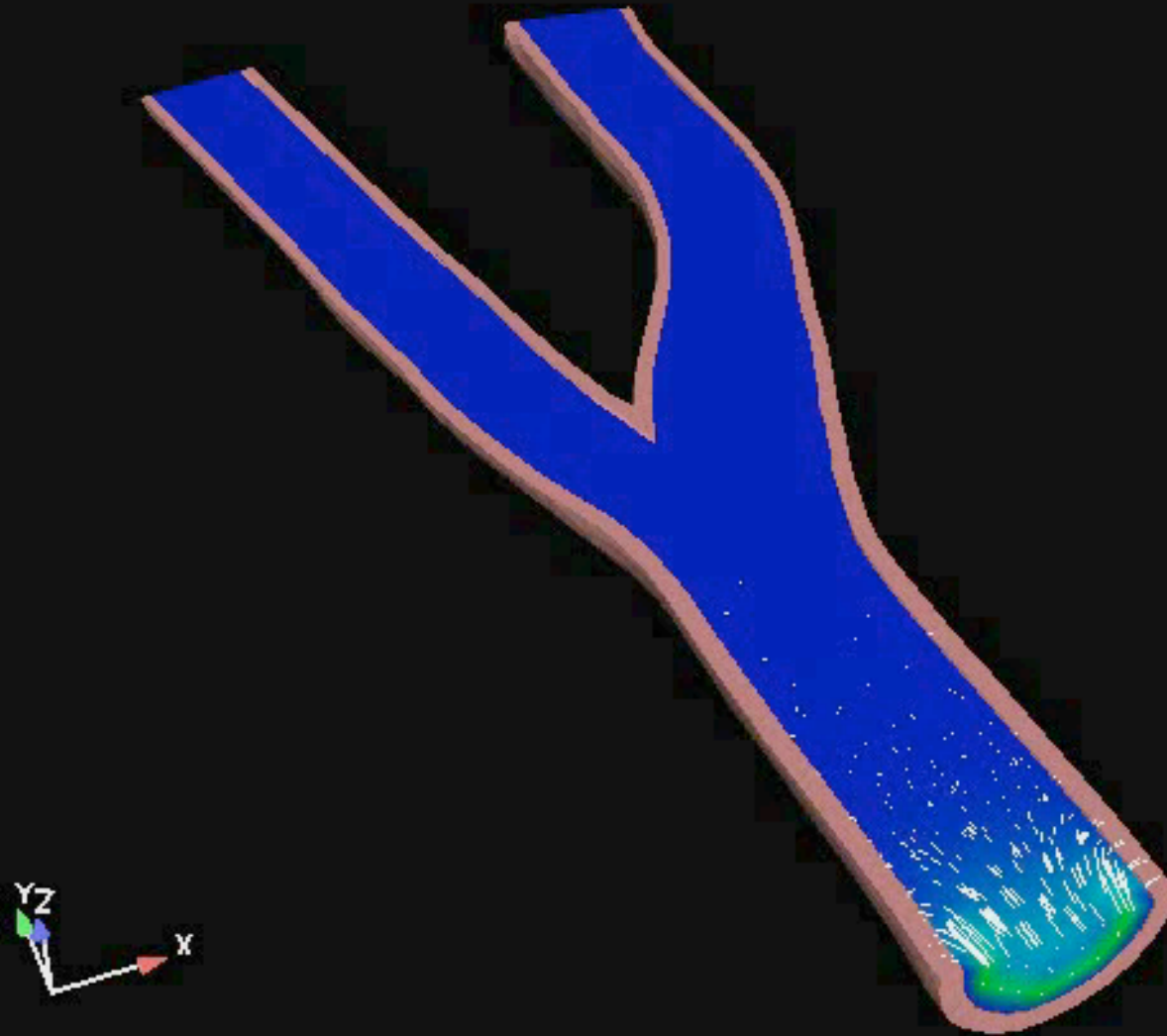
4. Clinical Applications

5. Virtual Surgery

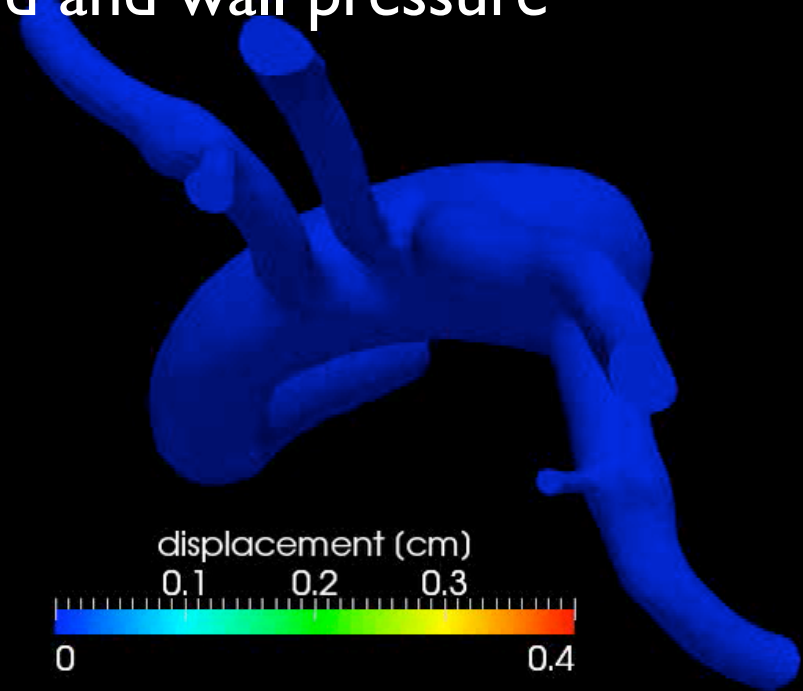
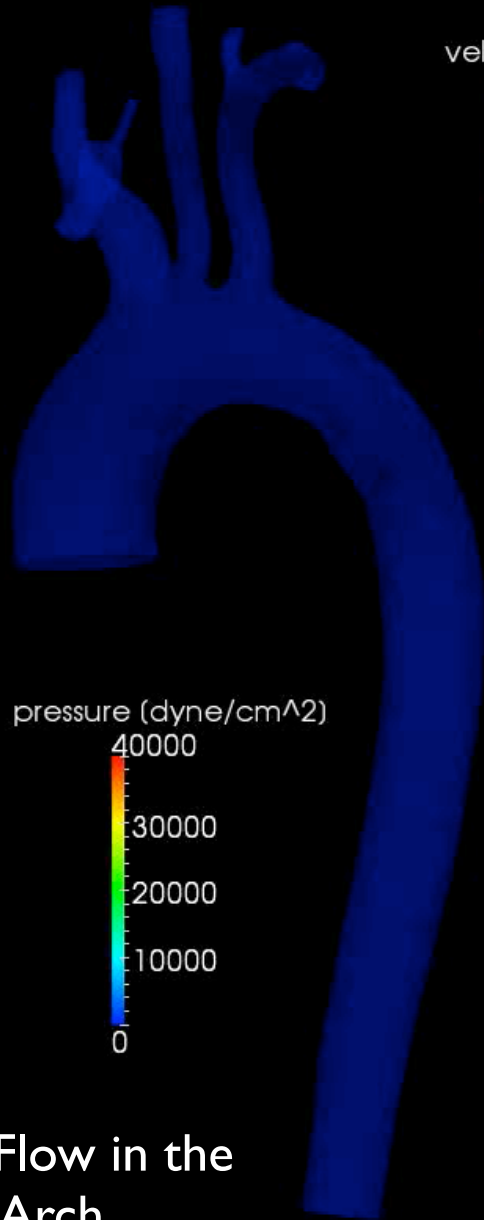
6. Prosthetic Design



Carotid wall deformation

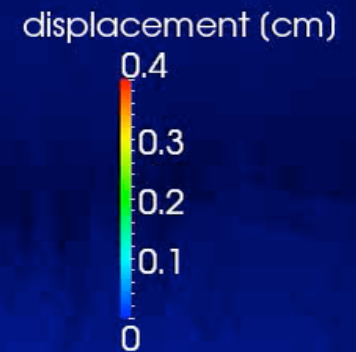


Aorta modeling: flowfield and wall pressure

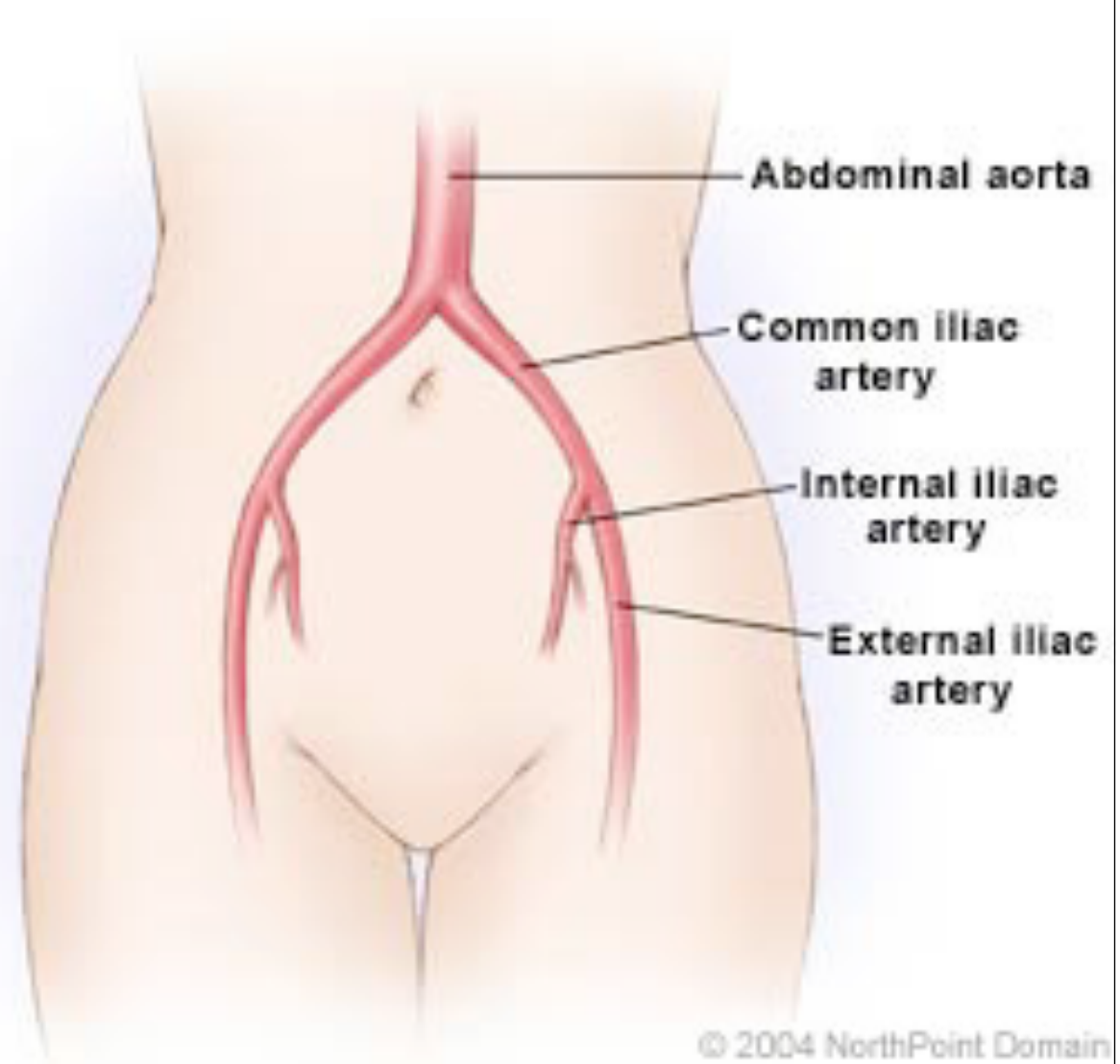
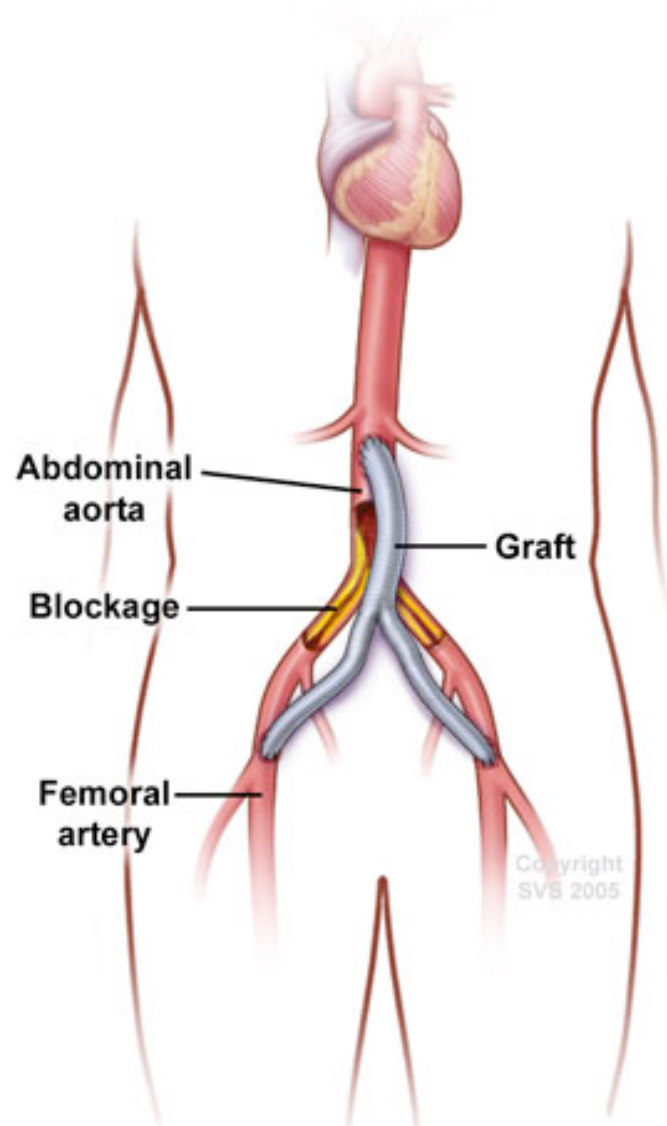


Blood Flow in the
Aortic Arch,
2 Heartbeats

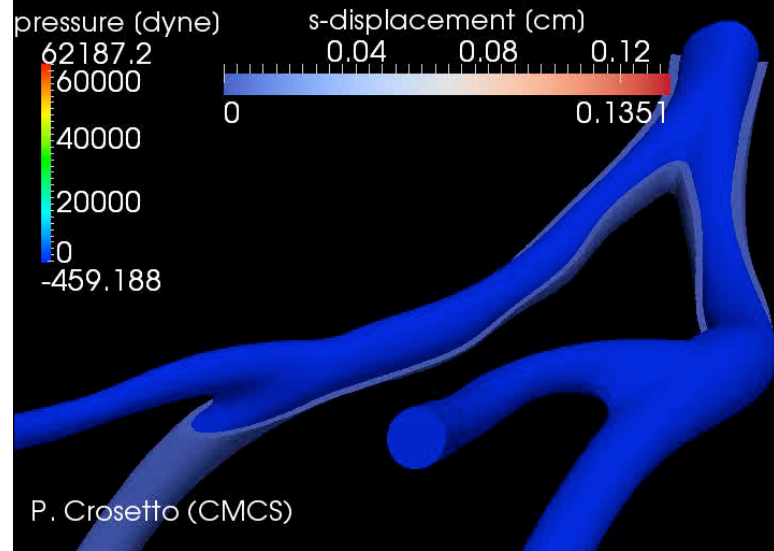
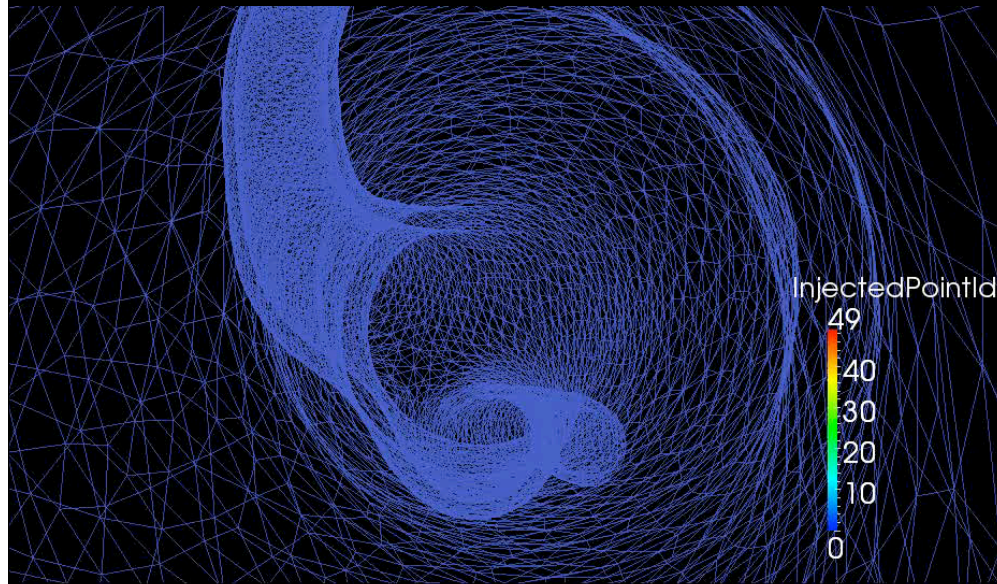
54 Time: 0.000



Iliac bifurcation modeling: flowfield and wall pressure



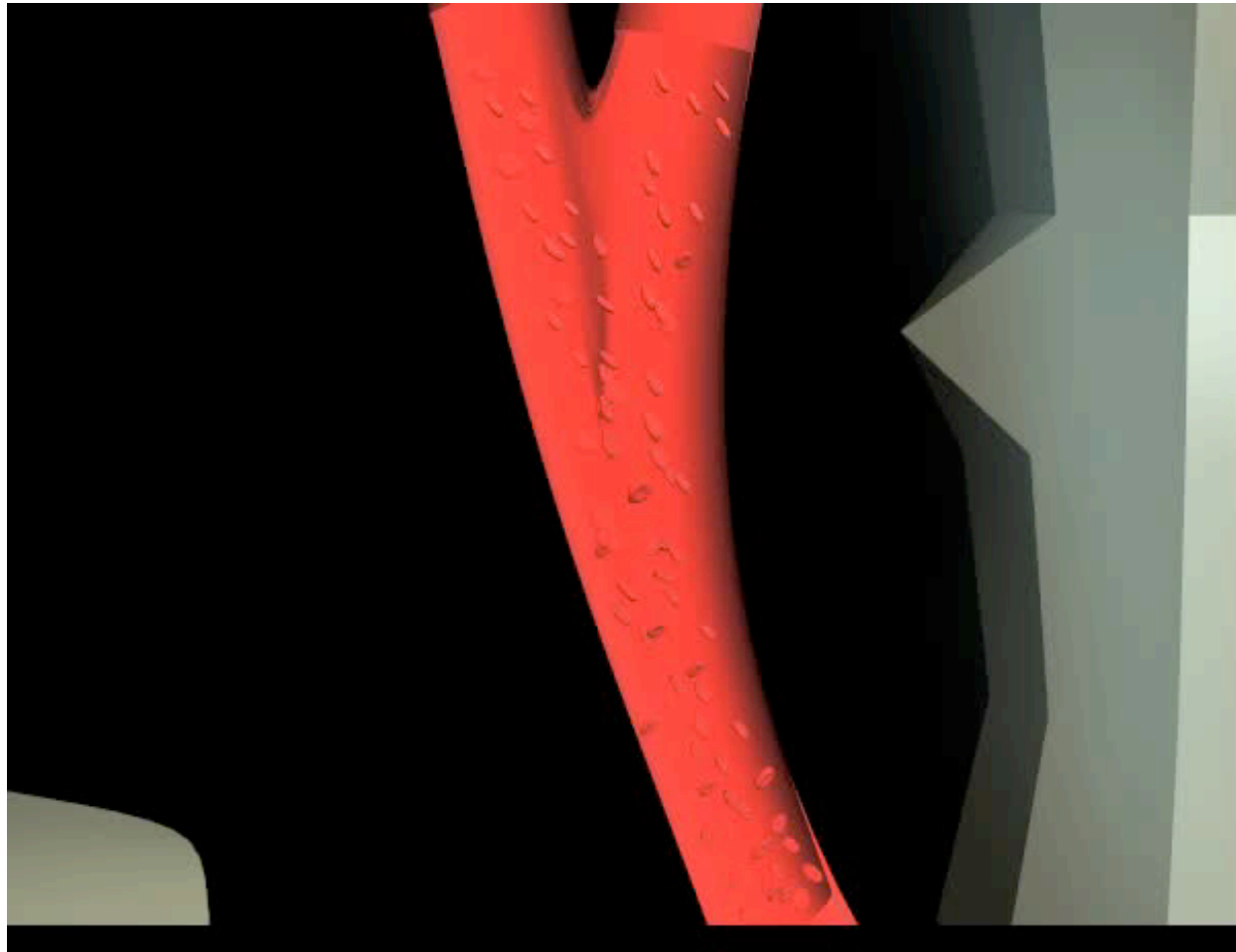
Iliac bifurcation modeling: flowfield and wall pressure



LifeV



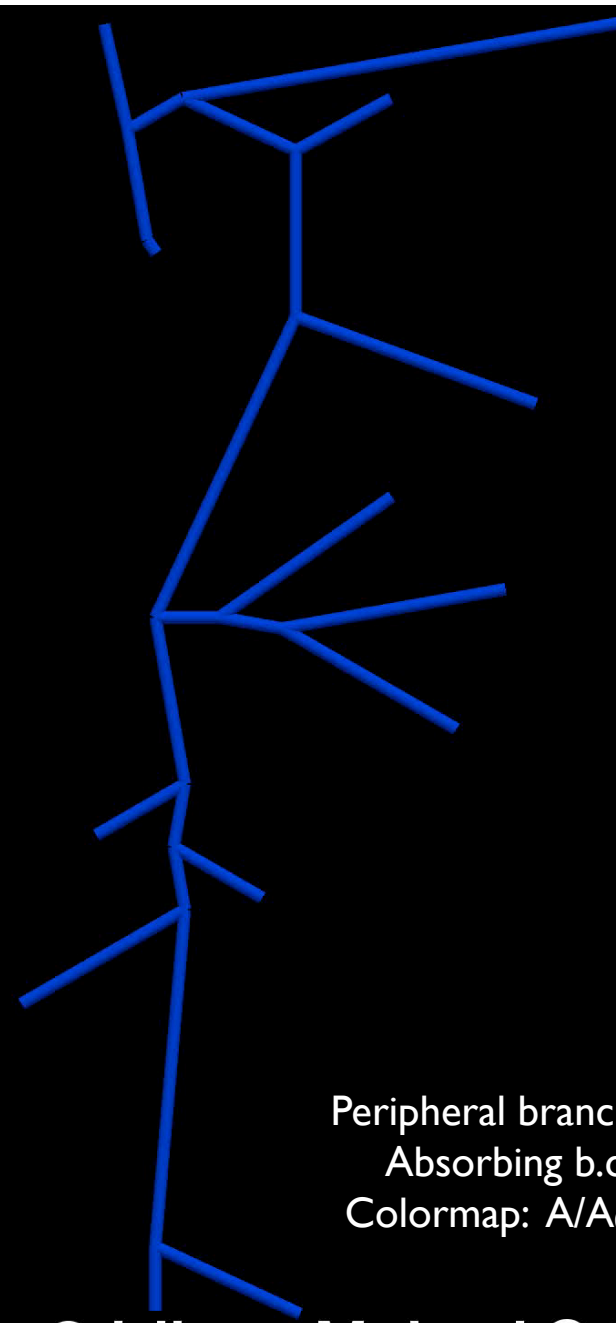
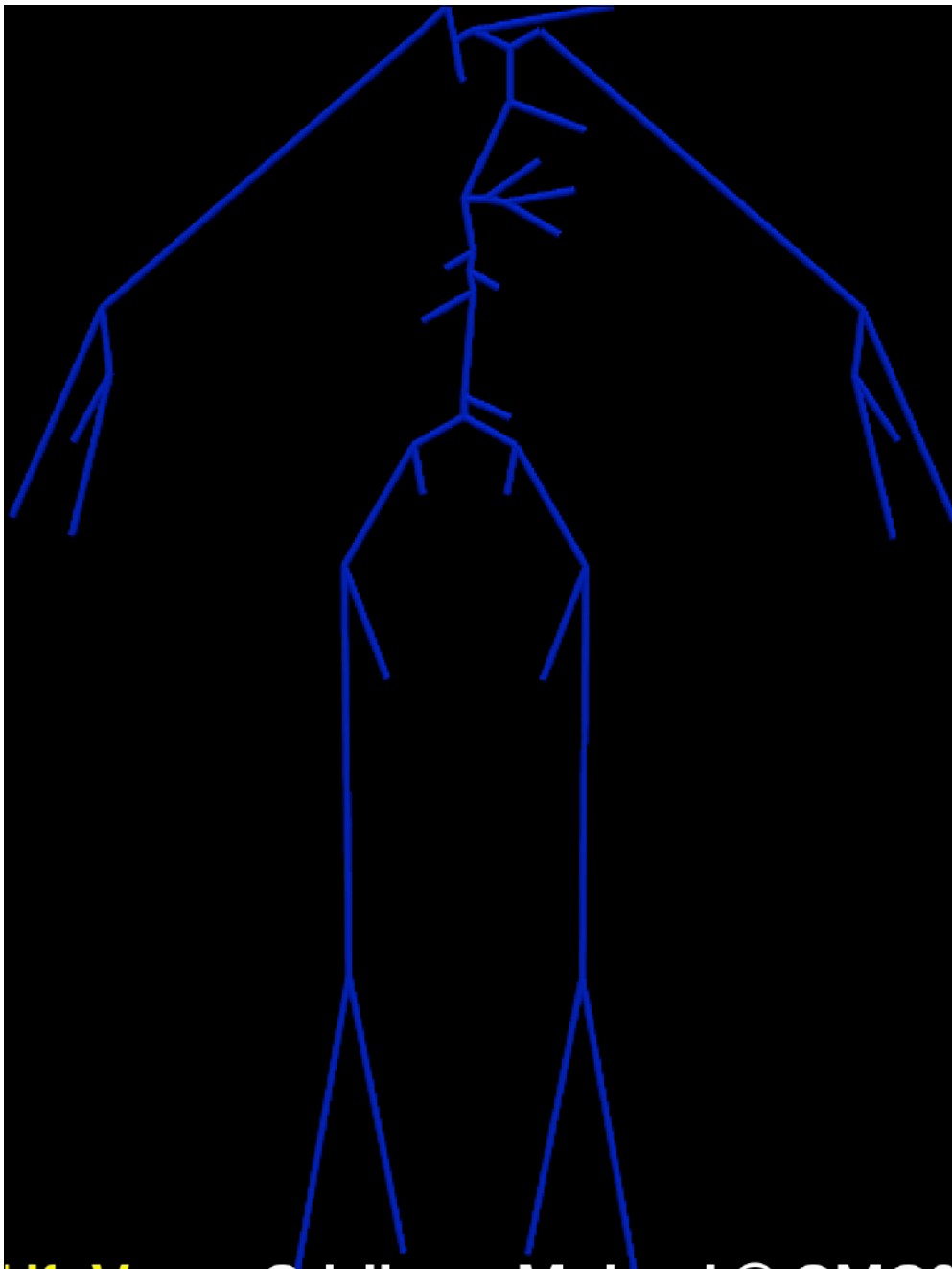
A geometric multiscale approach for the circulatory system



Local (level 1): 3D flow model (FSI between NS equations and elastodynamic eqs)

Global (level 2): 1D network of major arteries and veins (Euler hyperbolic system)

Global (level 3): 0D capillary network (DAE system)



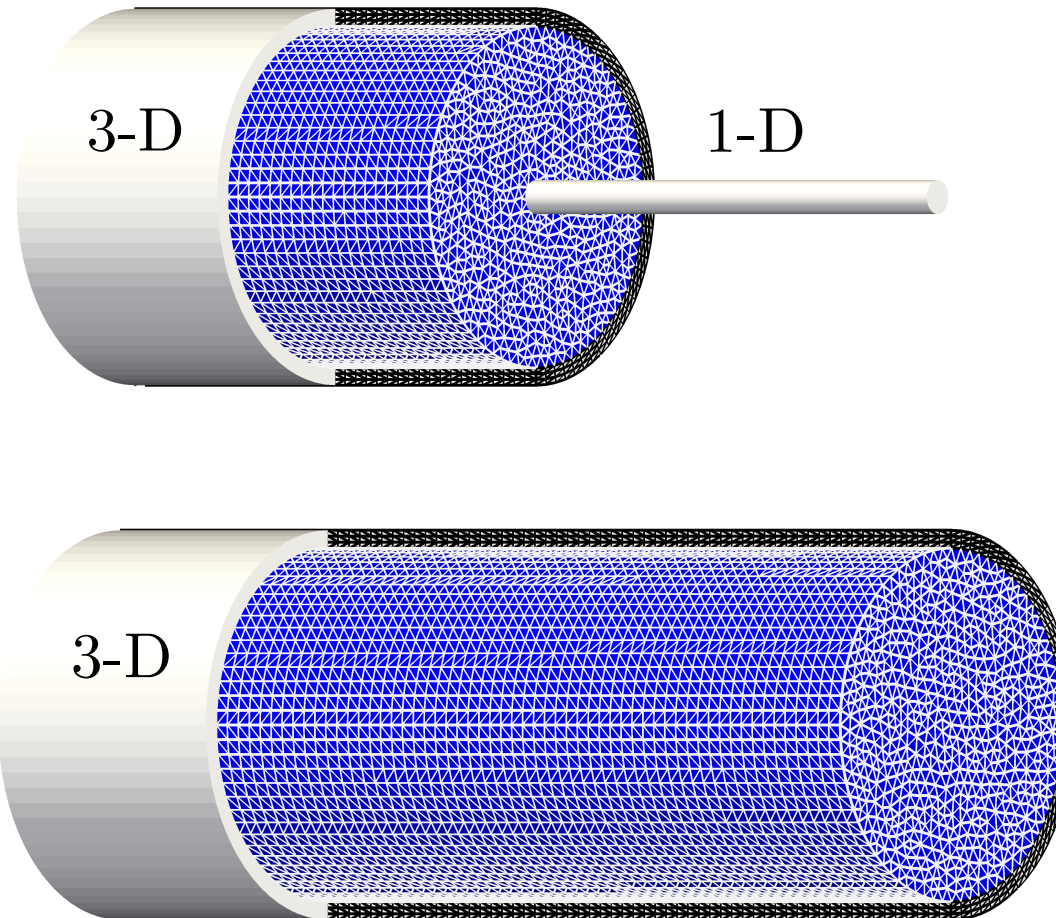
Peripheral branches:
Absorbing b.c
Colormap: A/A_0-1

LifeV

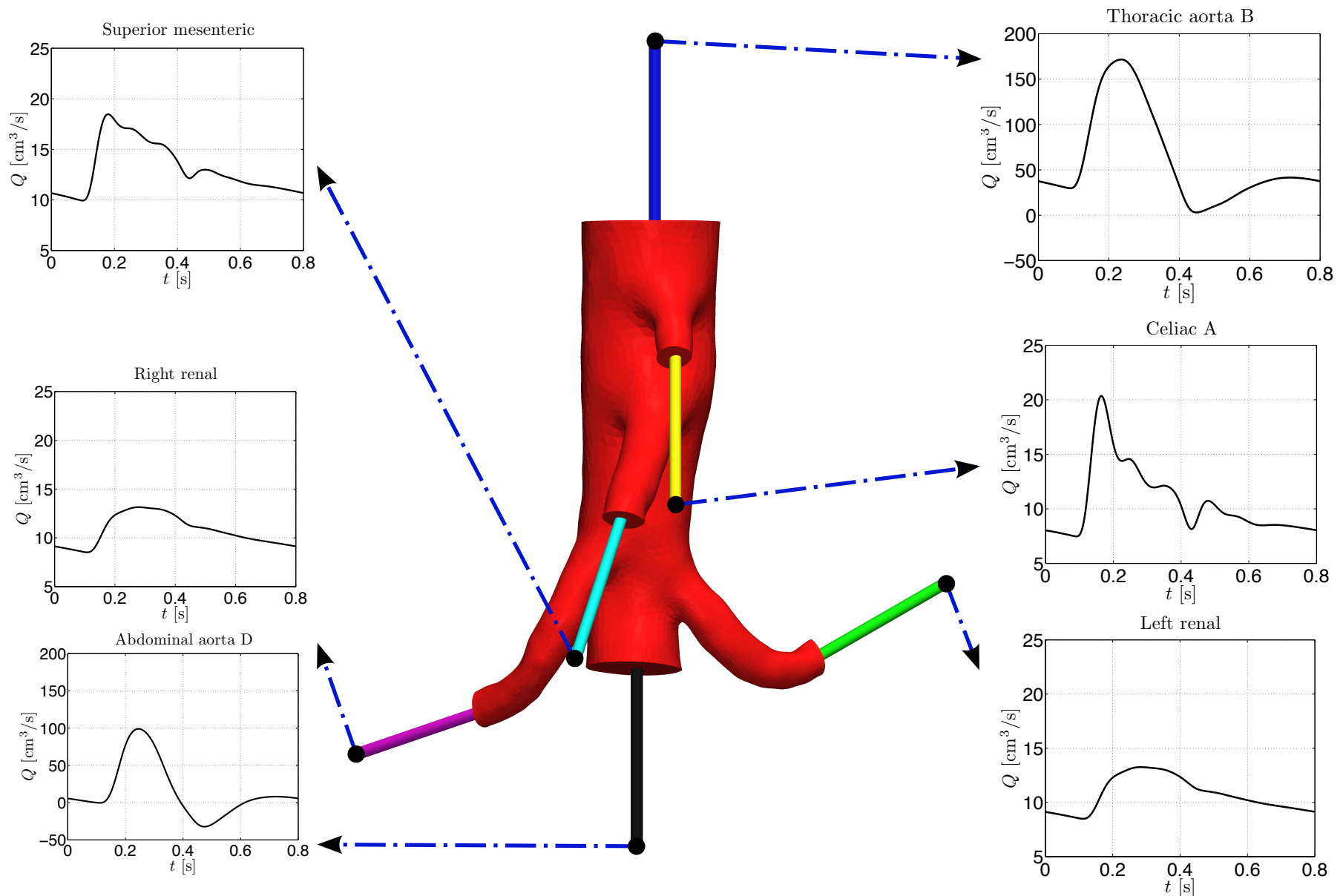
Cristiano Malossi @ CMCSLifeV

Cristiano Malossi @ CMCS

A geometric multiscale approach for the circulatory system



A geometric multiscale approach for the circulatory system



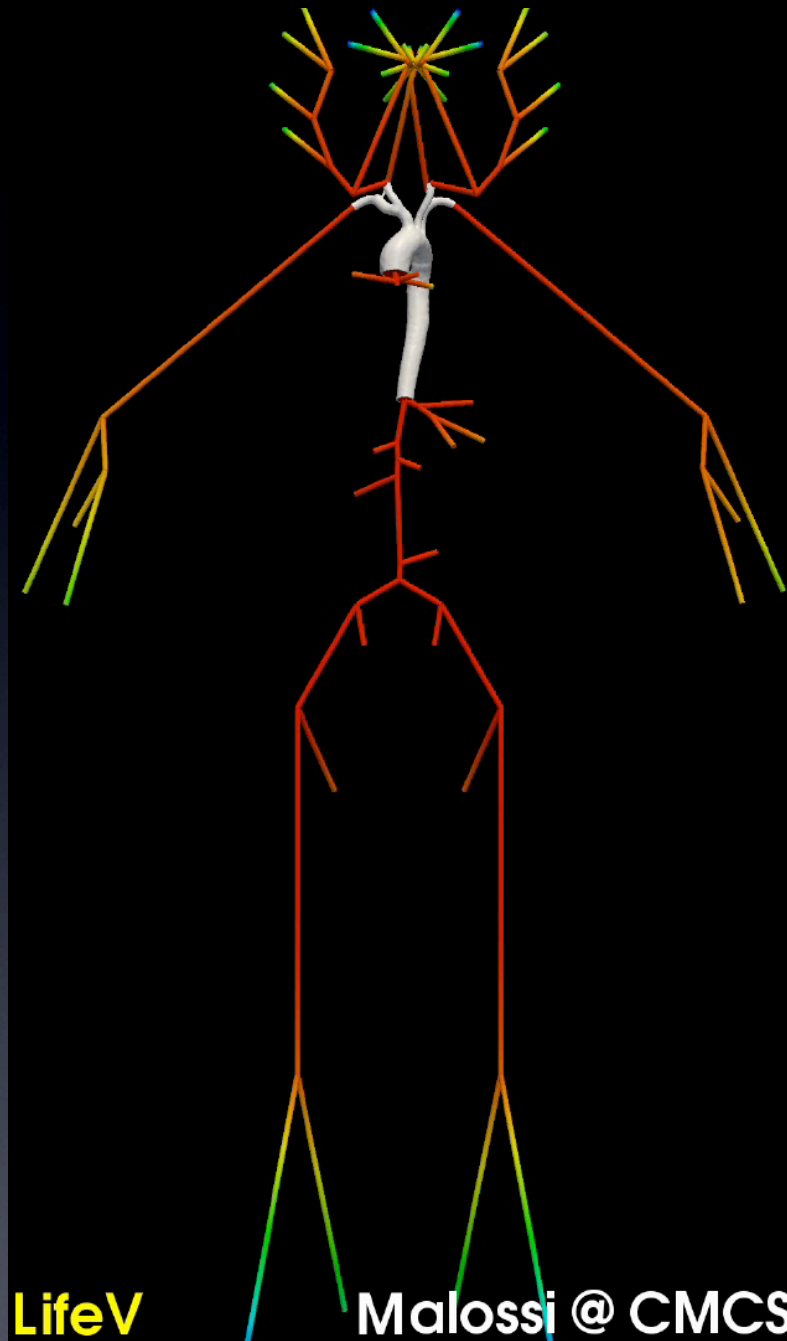
Geometric Multiscale - The Aortic Arch

Models:

- ◆ 3-D FSI Aorta
- ◆ 1-D arterial tree
 - 92 tapered elements
 - viscoelastic wall
- ◆ 0-D terminals
 - 47 Windkessel elements (RCR)

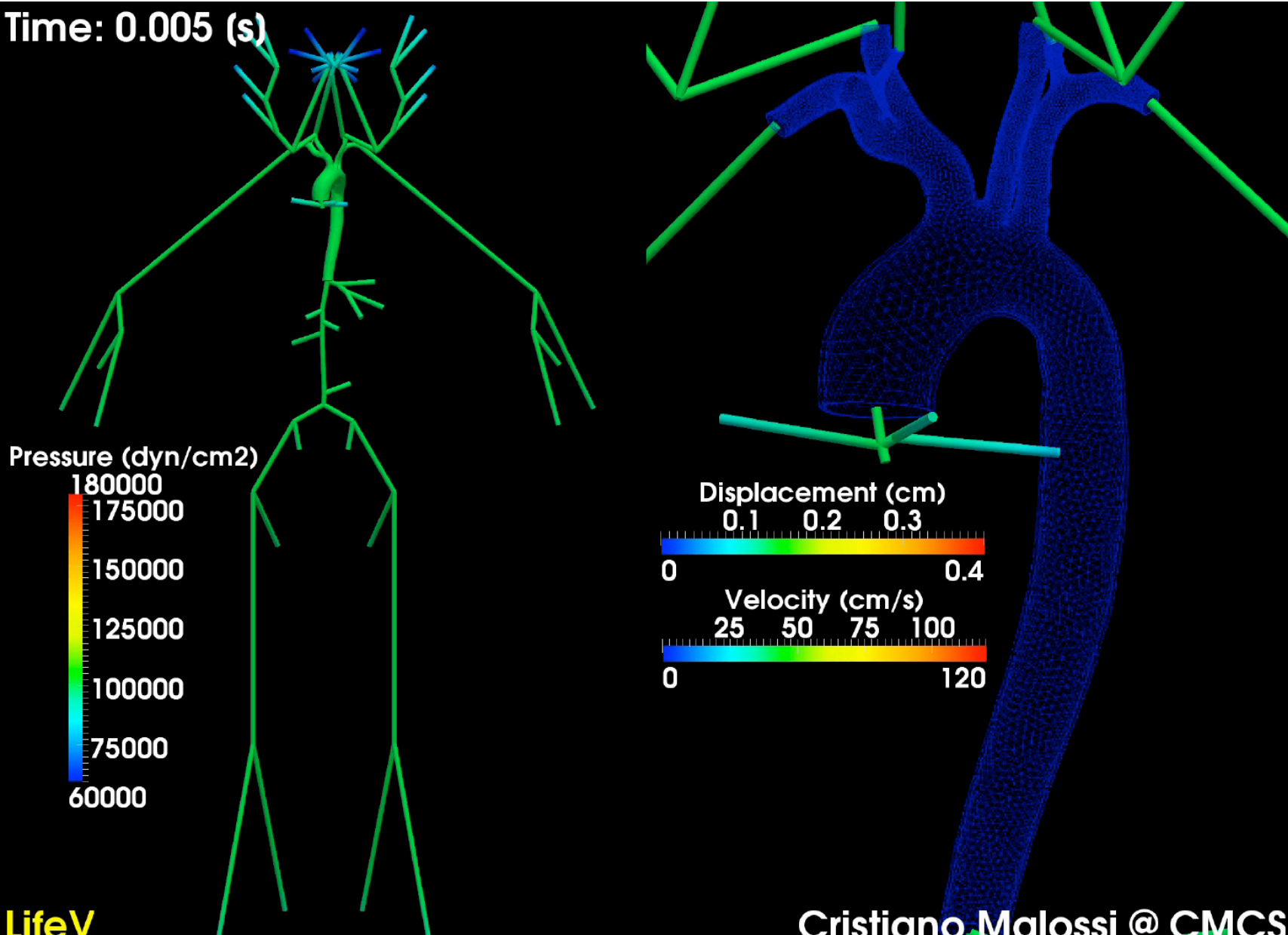
Coupling:

- ◆ averaged/integrated quantities at the interfaces (flow rate or normal stress)
- ◆ segregated approach for the solution of the coupled problem (Newton, inexact-Newton, or Broyden methods)



Flowfield in aortic branch and carotid arteries

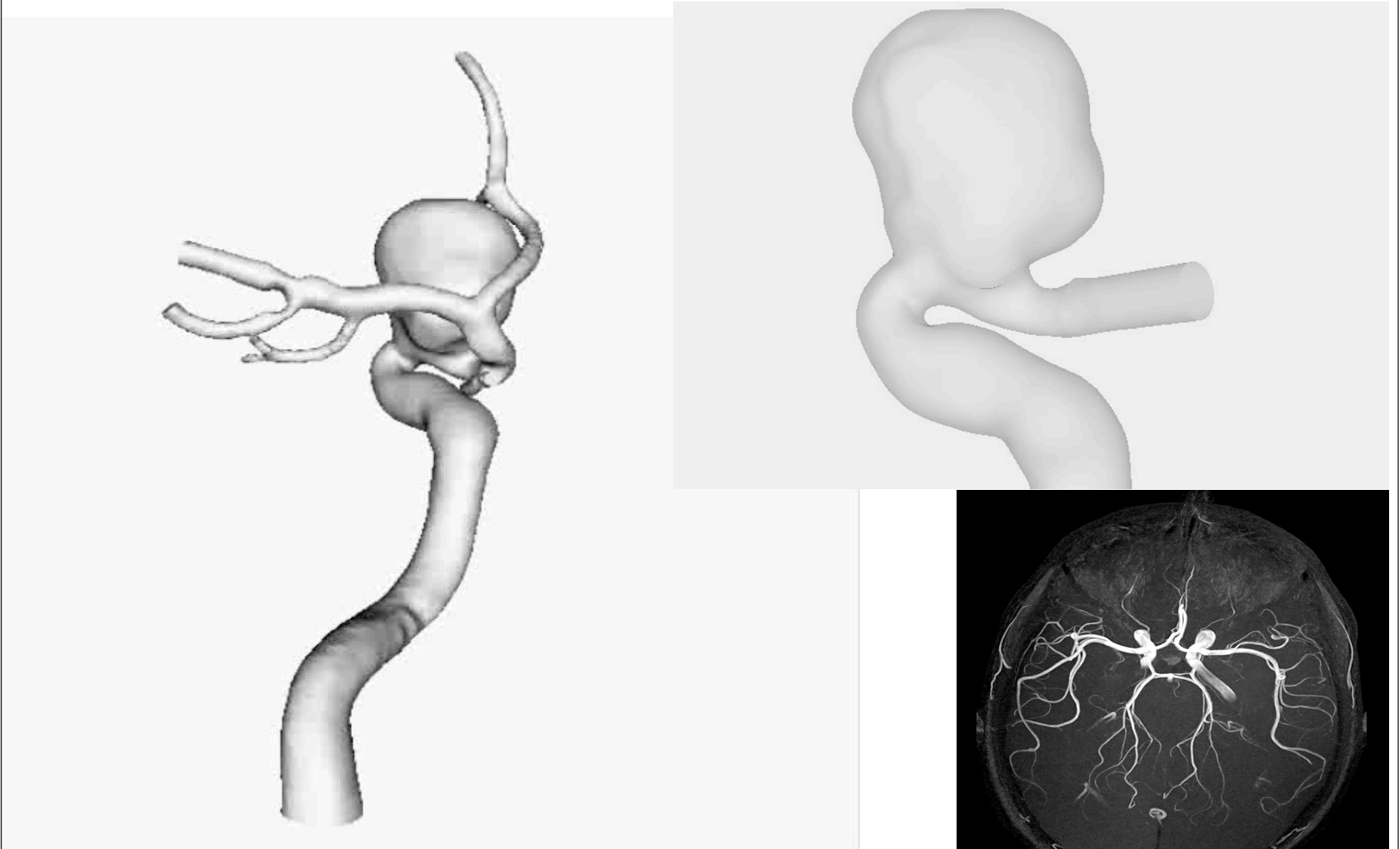
Time: 0.005 (s)



LifeV

Cristiano Malossi @ CMCS

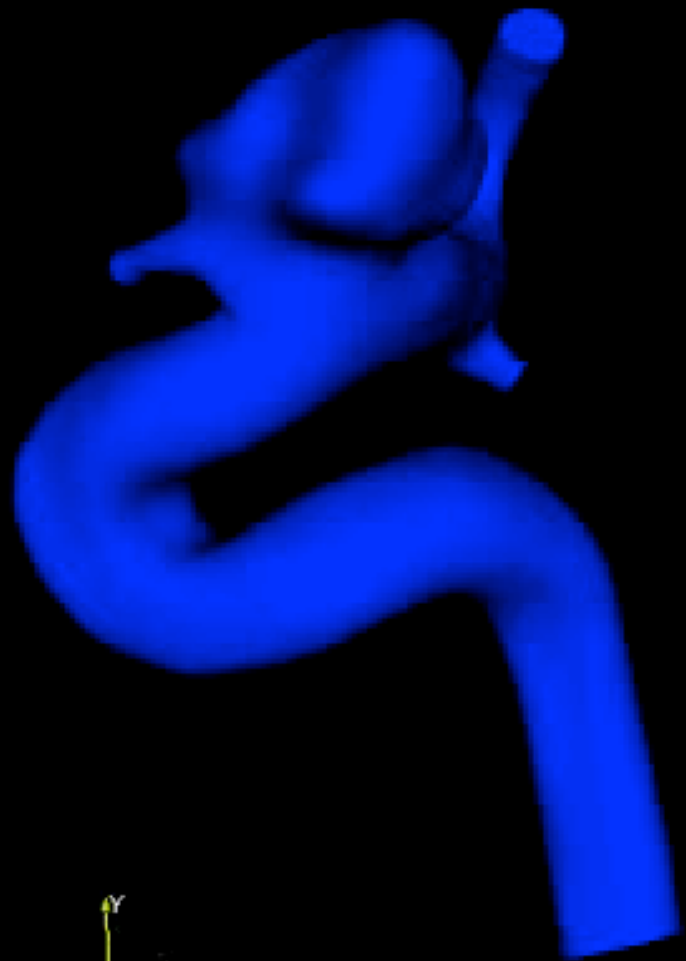
Flow in a cerebral aneurysm during a full cardiac pulse



Simulation carried out by T. Passerini

WSS - Wall Shear Stress

LifeV



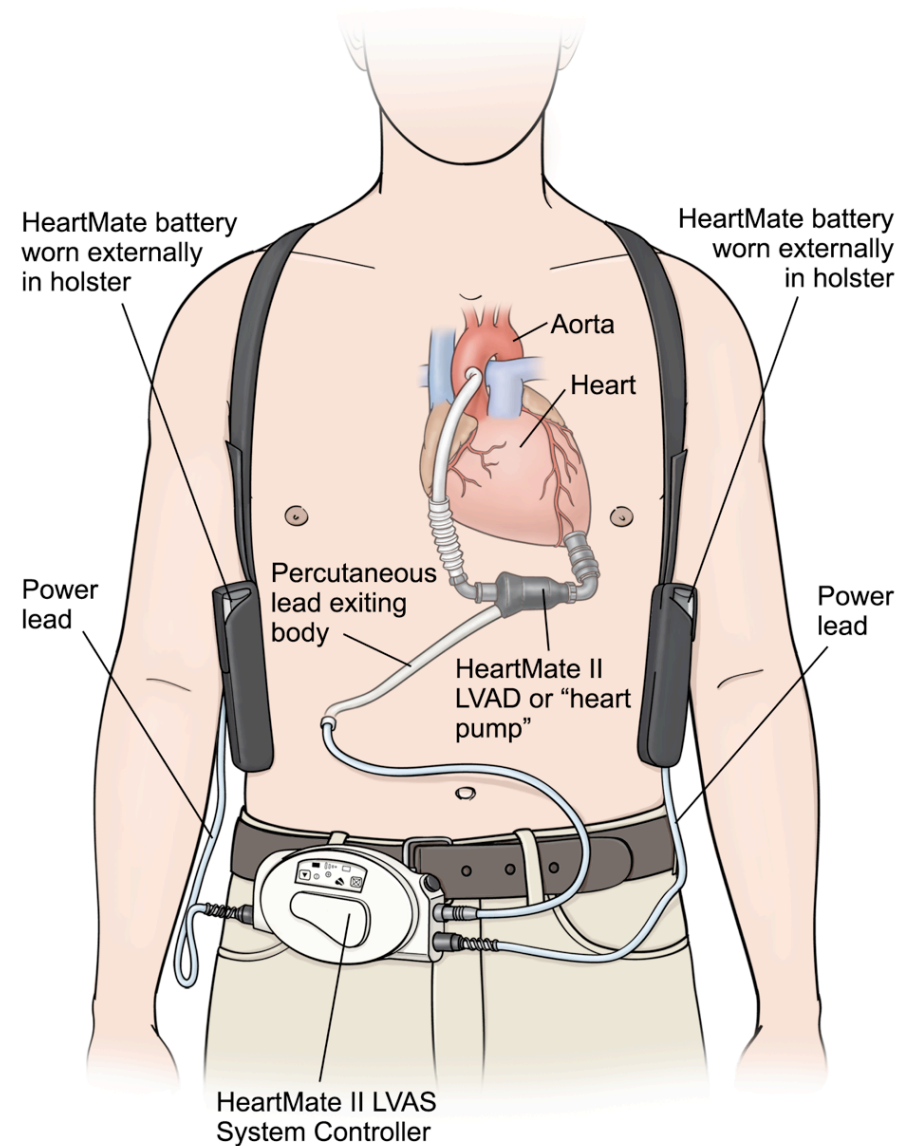
S. Deparis @ cmcs-epfl

Time: 0.00

Prosthetic Devices - saving human lives

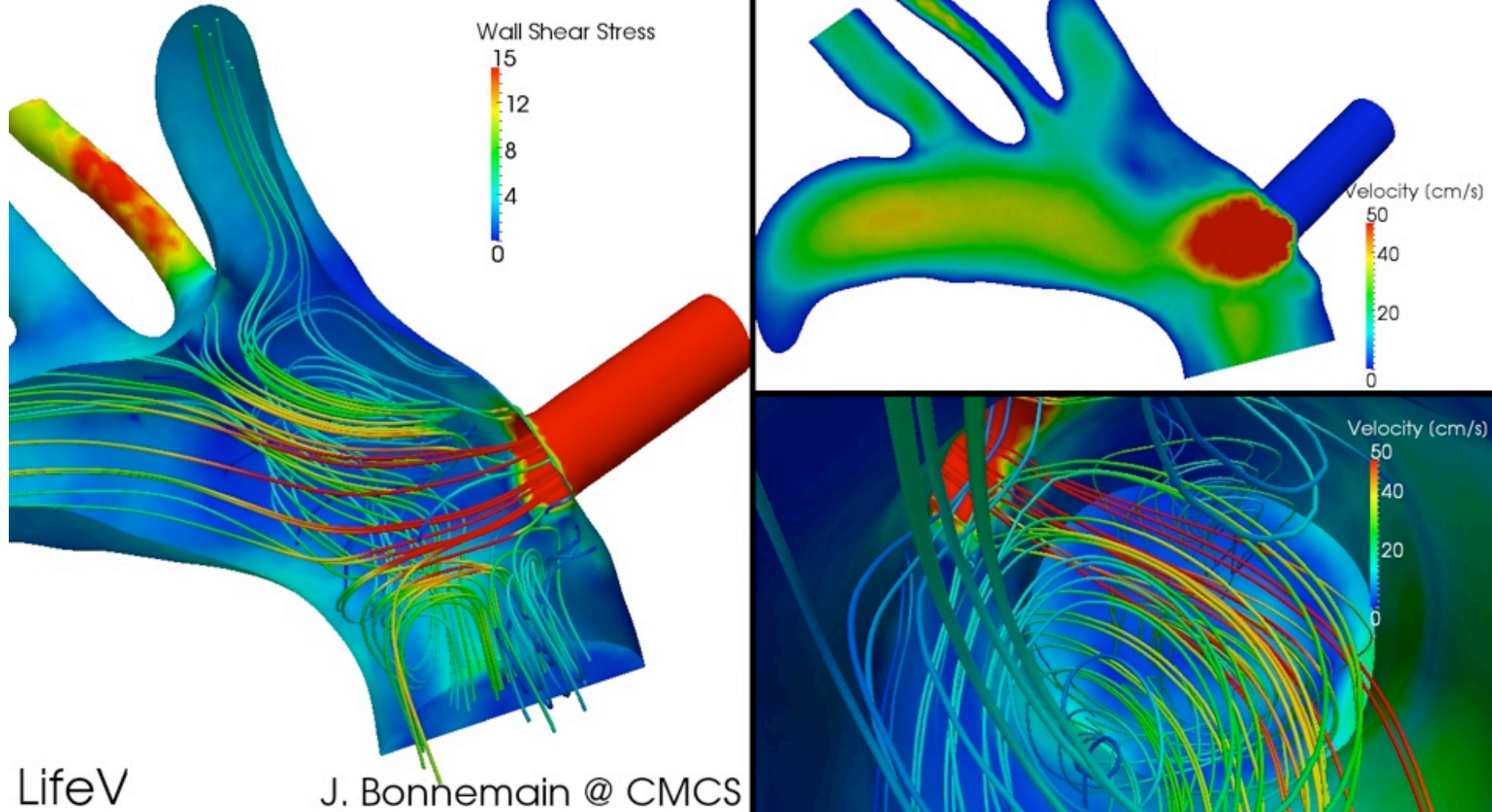
Ventricular Assisted Device (VAD)

- Treatment of terminal heart failure
- Left ventricle and / or right ventricle assistance
- For patients awaiting heart transplantation
- Also for patients ineligible for transplantation



Cannula of a Ventricular Assisted Device (VAD)

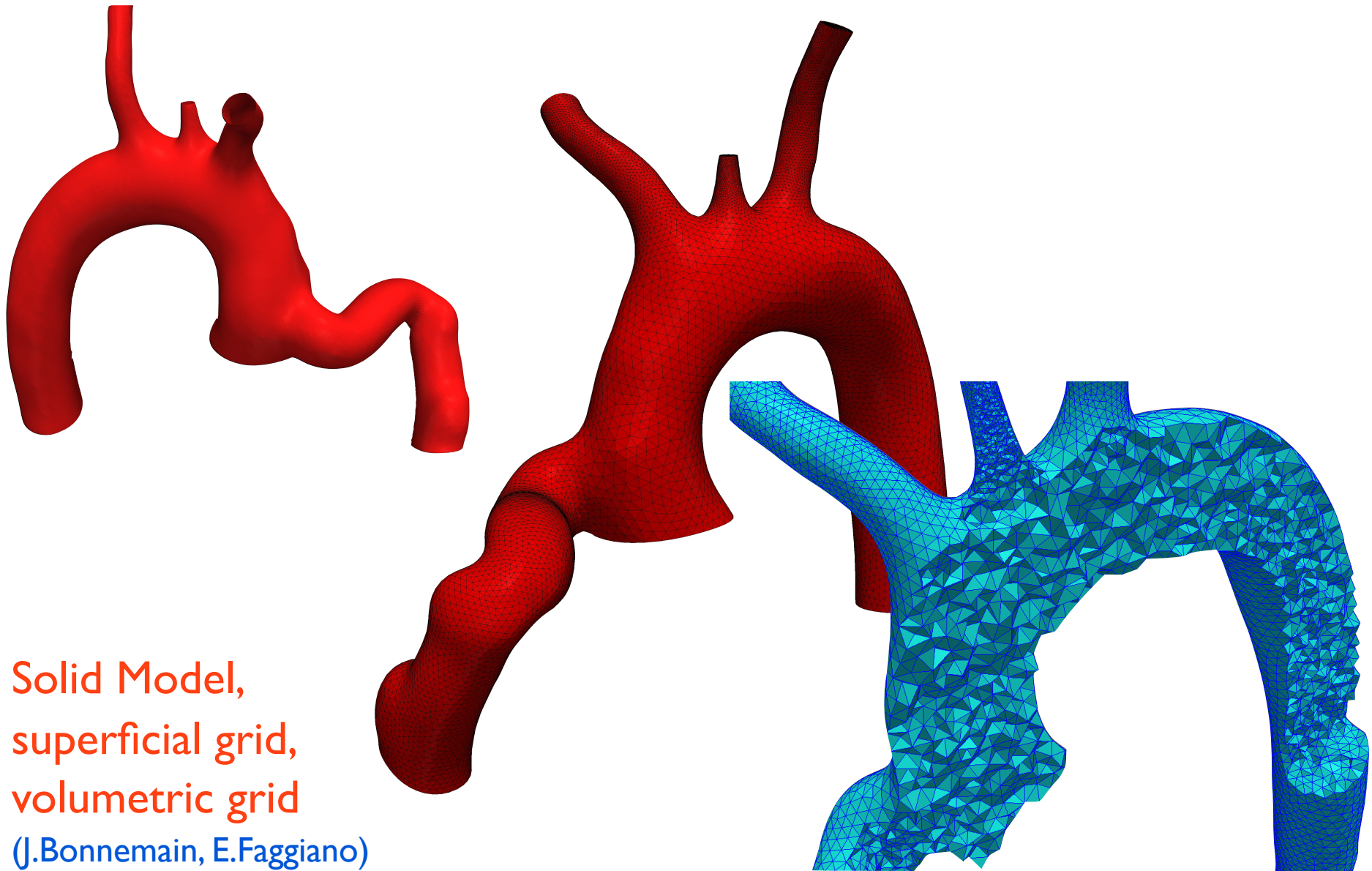
Time: 0.914 s.



LifeV

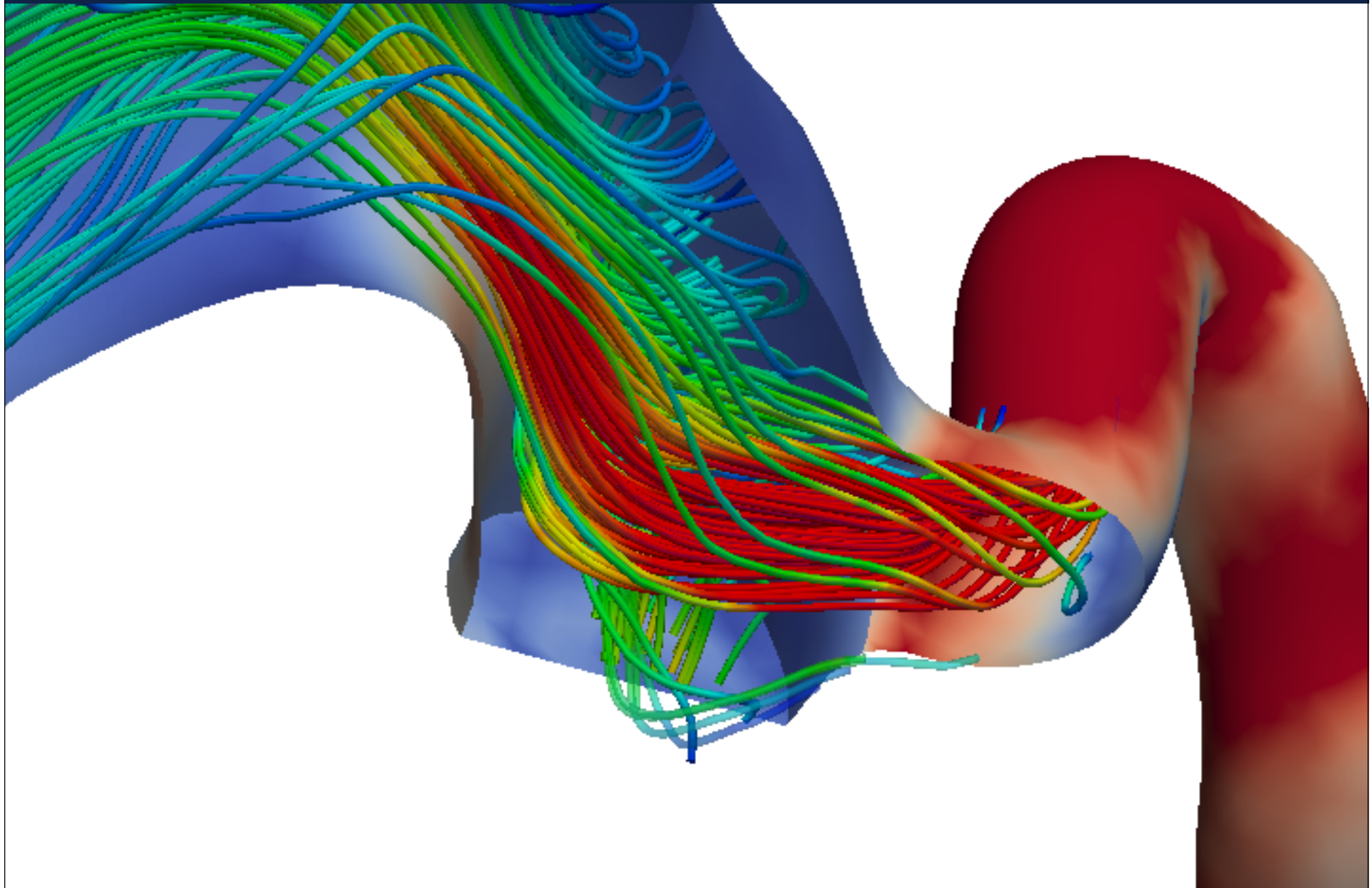
J. Bonnemain @ CMCS

Cannula of a Ventricular Assisted Device (VAD)

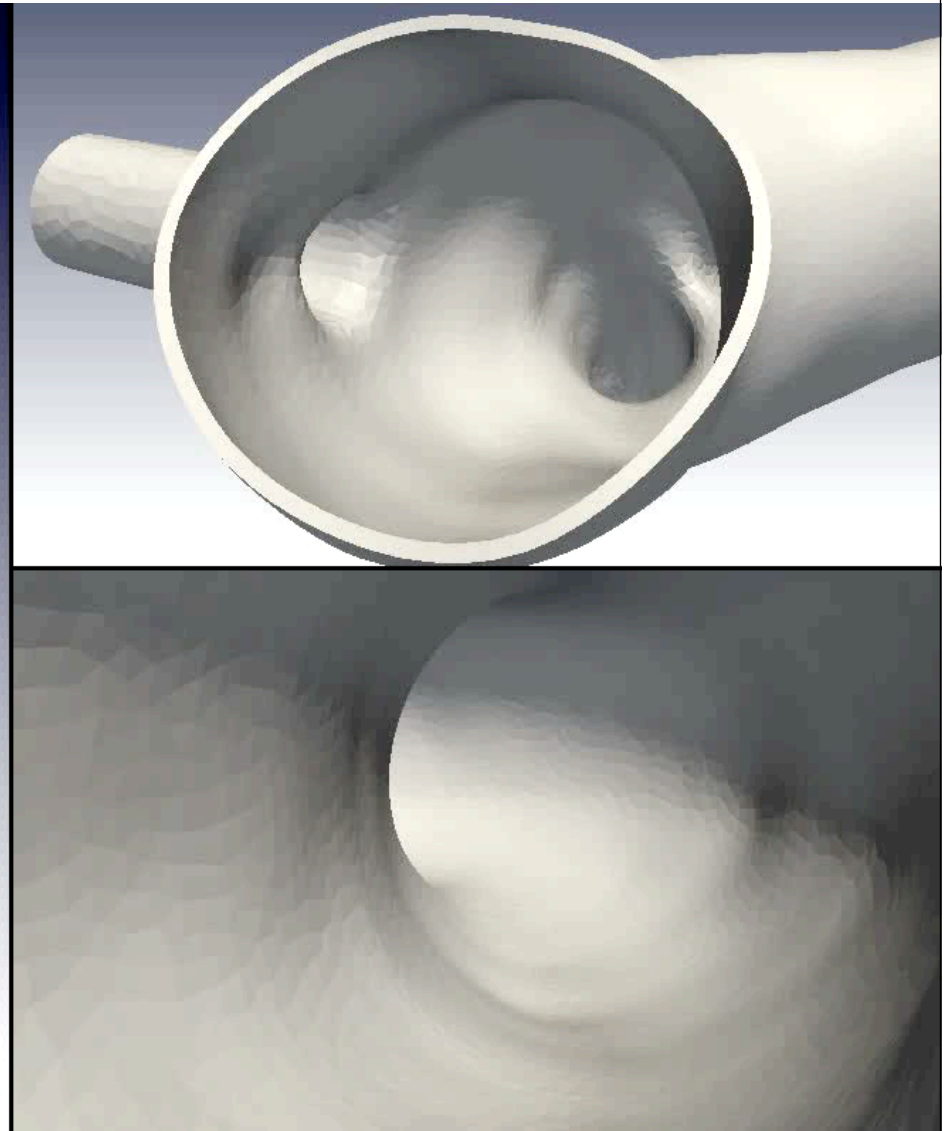
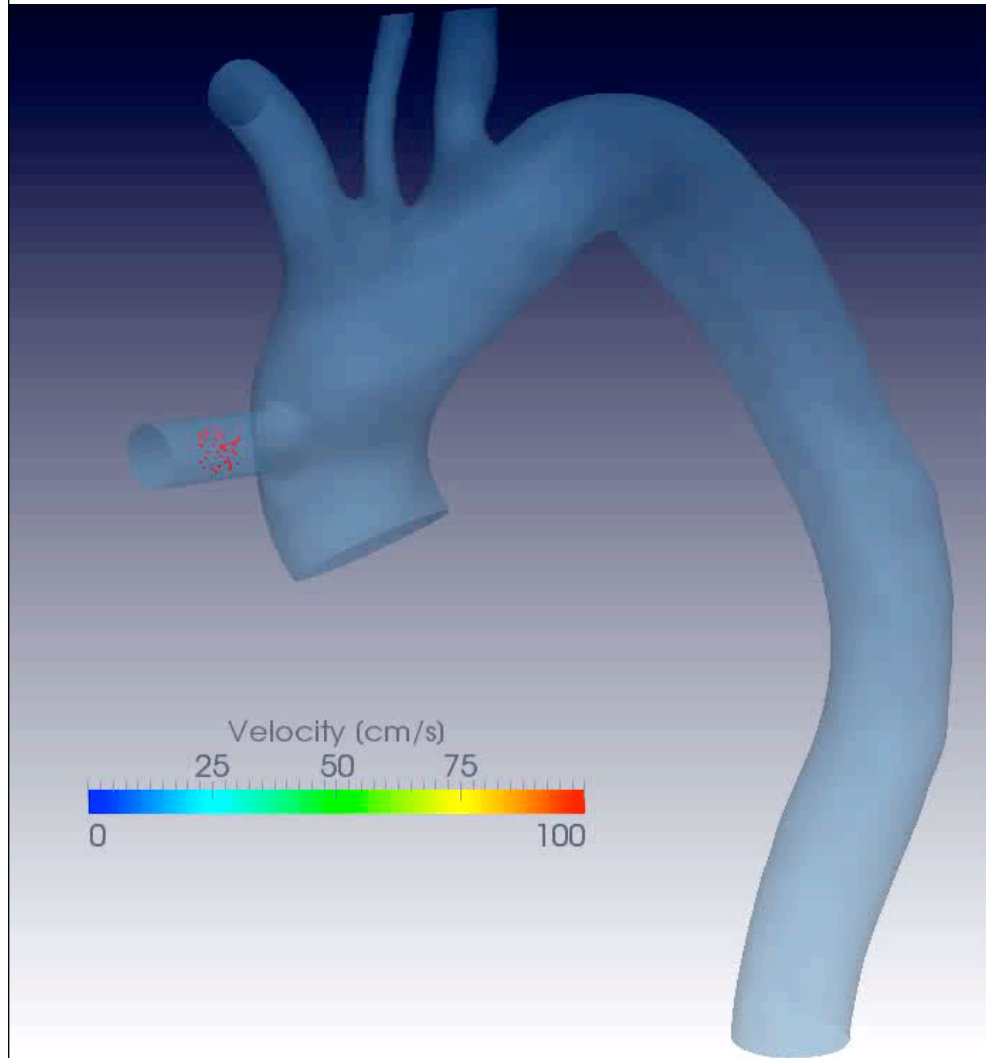


Solid Model,
superficial grid,
volumetric grid
(J.Bonnemain, E.Faggiano)

Cannula of a Ventricular Assisted Device (VAD)



Cannula of a Ventricular Assisted Device (VAD)

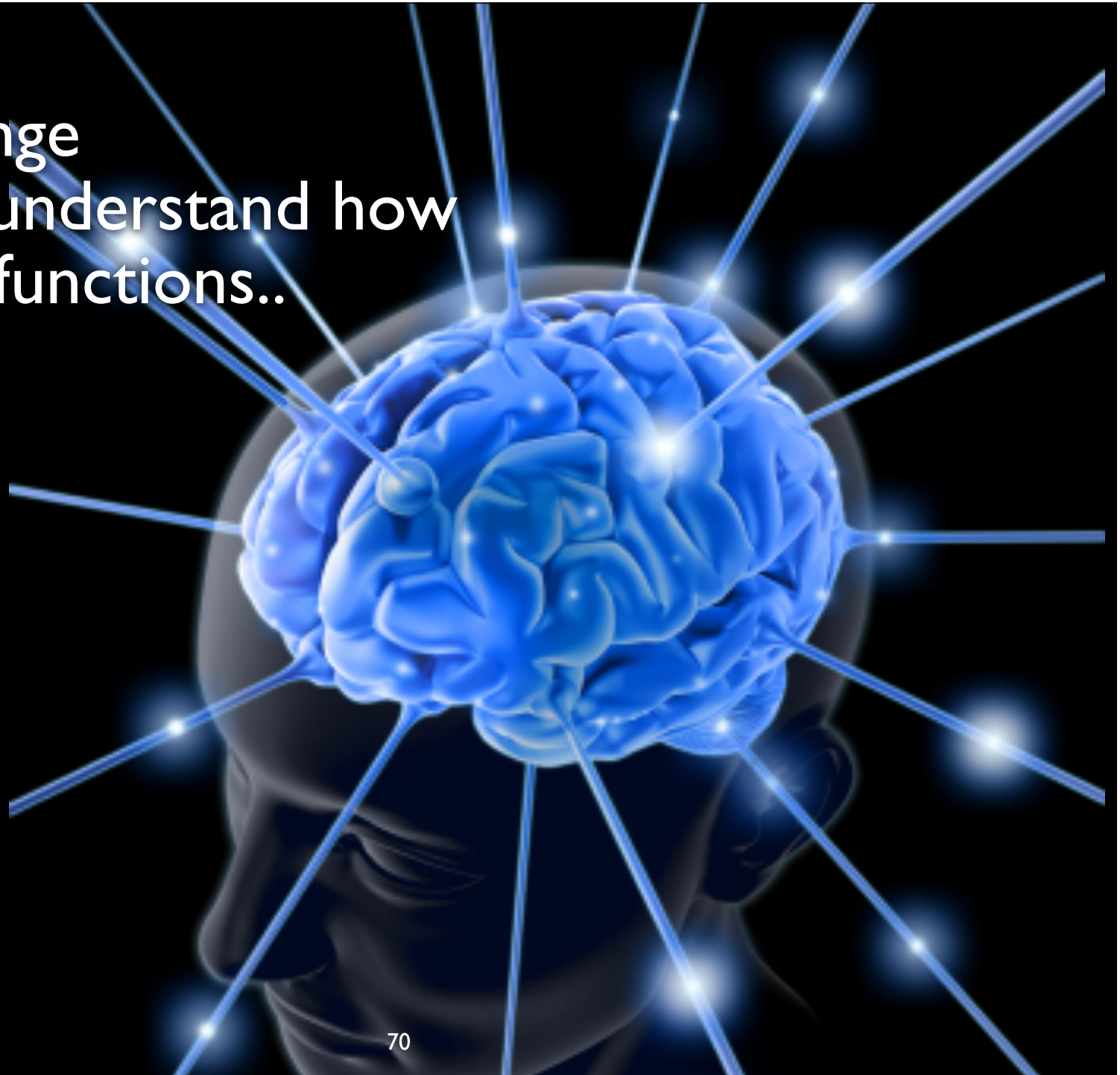


Time: 0.003 s.

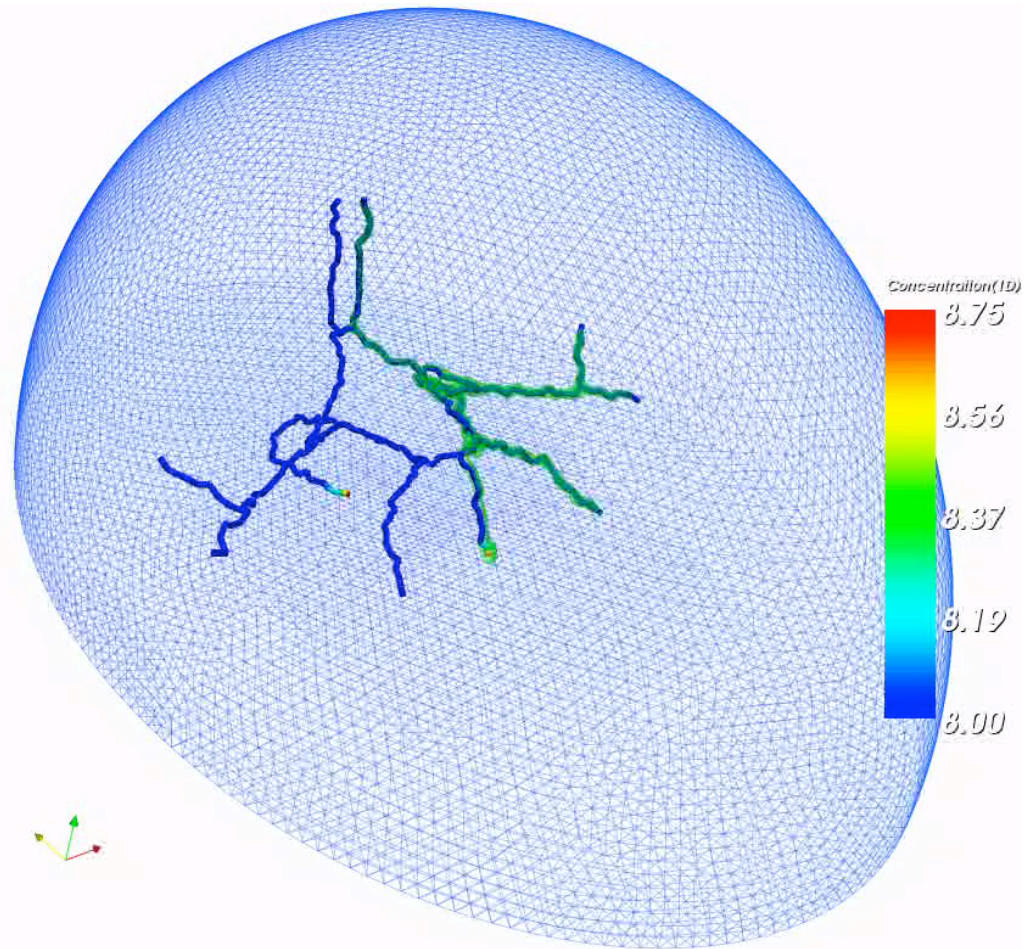
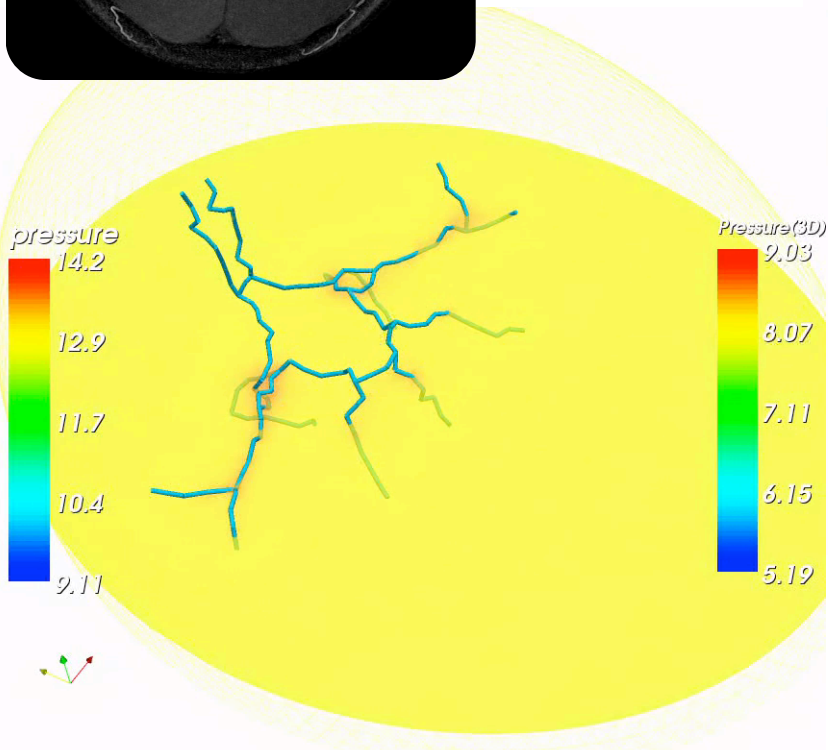
LifeV

J. Bonnemain @ CMCS

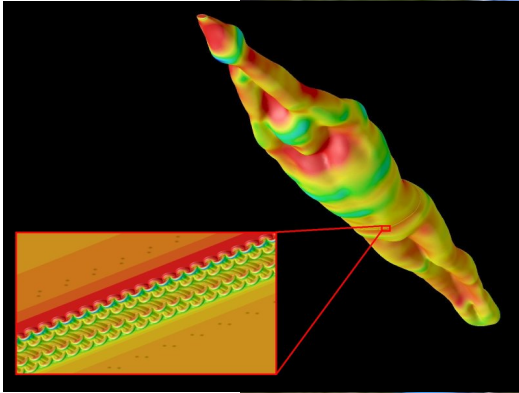
Math is
the challenge
to better understand how
our brain functions..



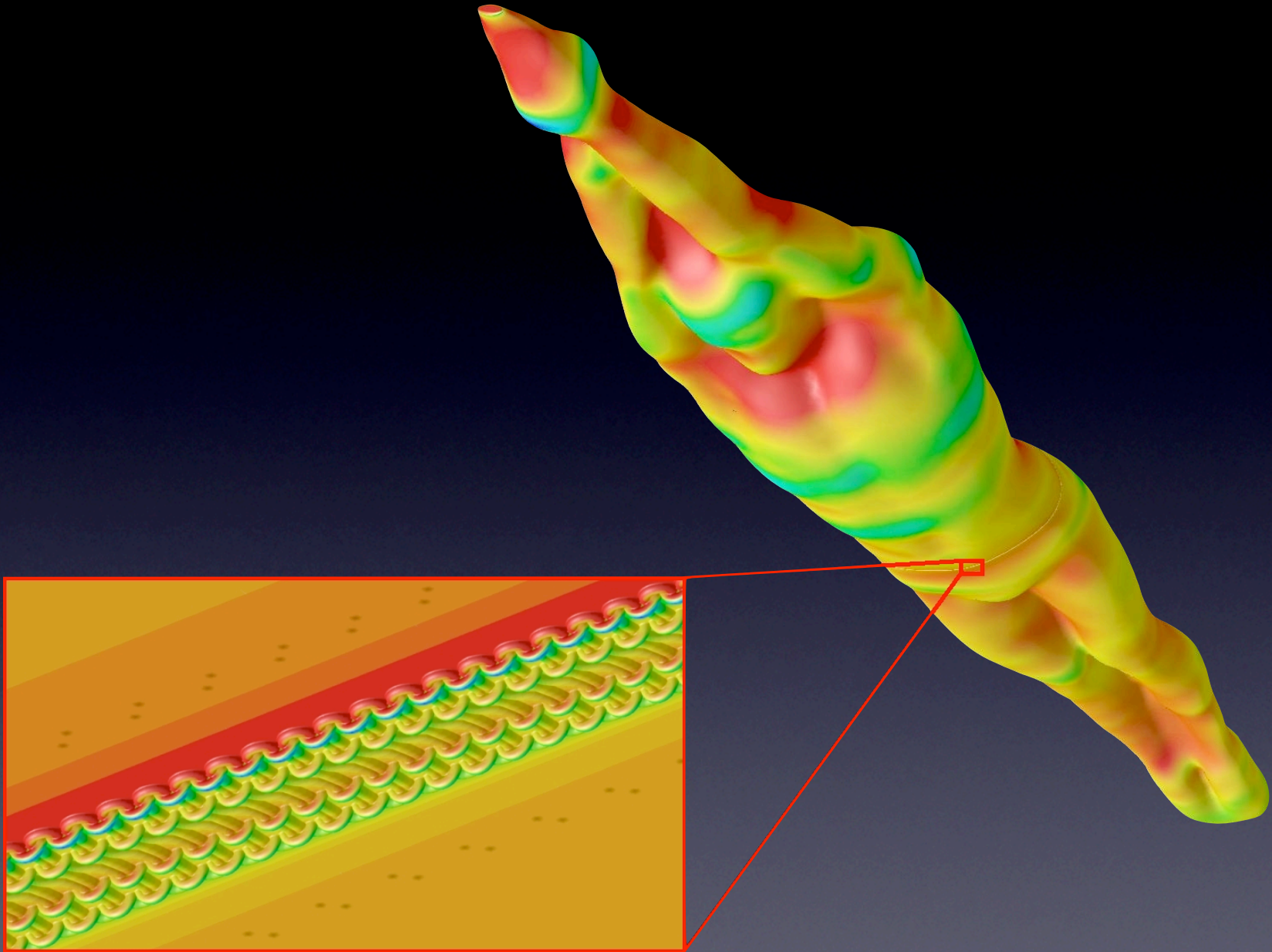
Oxygen Diffusion and Blood Pressure in the Brain



Real and virtual swimmers



(N.Parolini, A.Veneziani)



Racing cars: facts and figures

Performance due to:

15% engine

35% weight

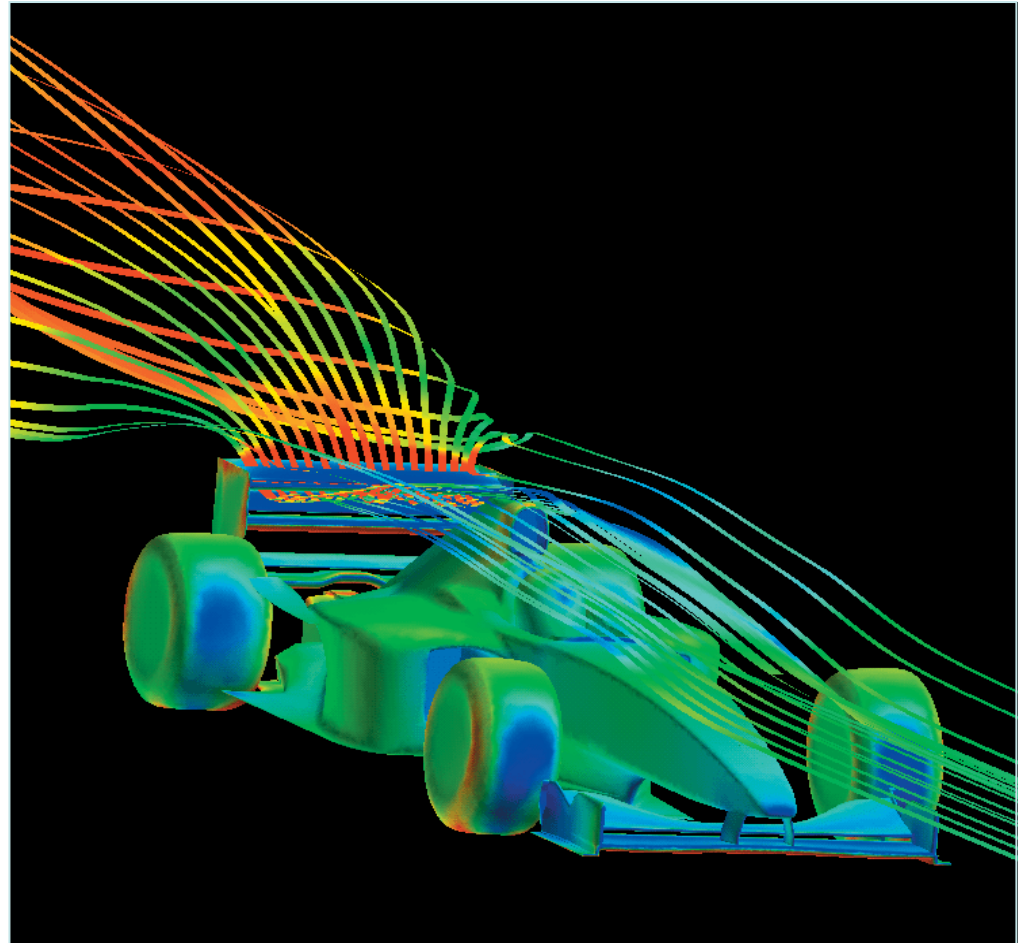
50% aerodynamics

Time:

8 months for virtual design

(virtual models are built up through CFD-CSD-Vehicle dynamics facts and behaviors are understood through wind tunnel experimental quantities are measured)

1 month for assembling



Math is our Lab

- cheap
- fast
- optimal

The SOLAR IMPULSE Airplane



Math Modelling and AMERICA'S CUP



(N.Parolini, D.Detomi)

AC31 - 2003



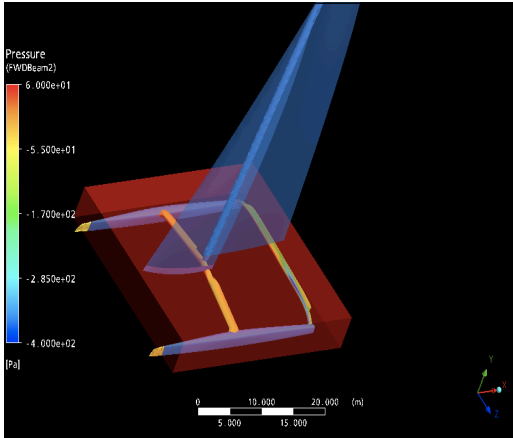
(G. Cowles, M. Sawley, N. Parolini, A. Q₈)

AC32-2007



(N.Parolini, D.Detomi, A.Q.)

AC33-2010



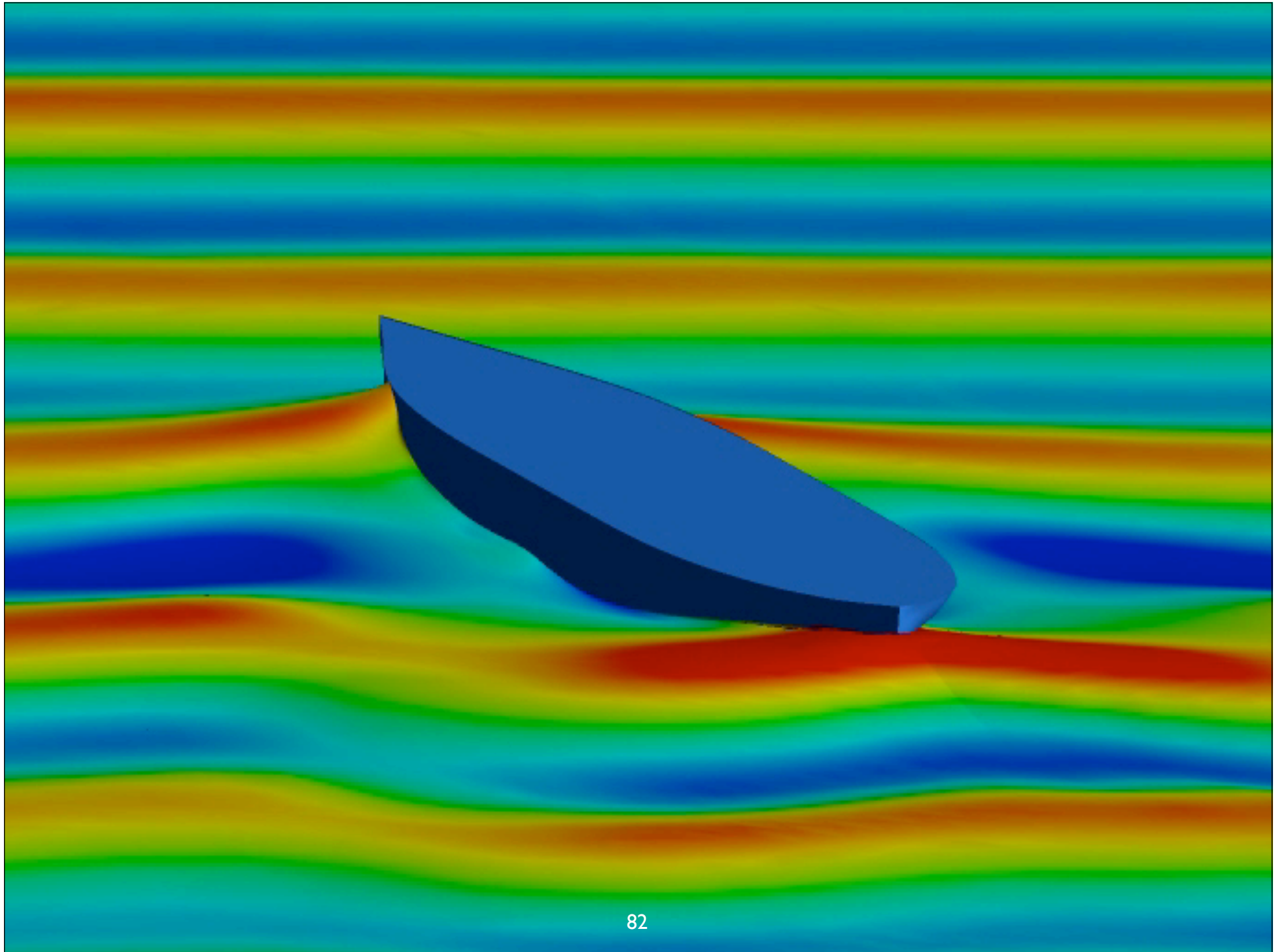
AC 2010
ALINGHI 5
the Multihull

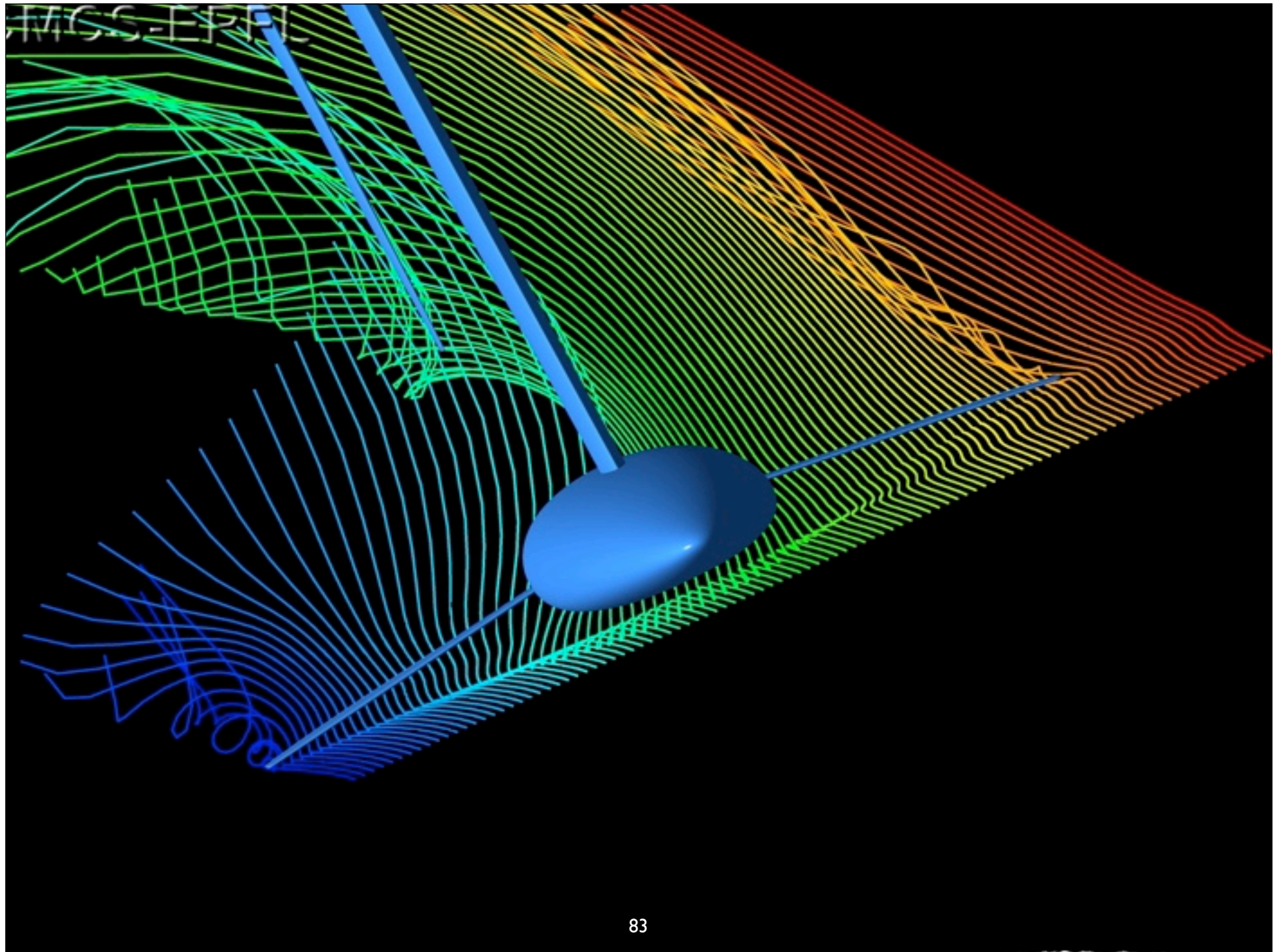


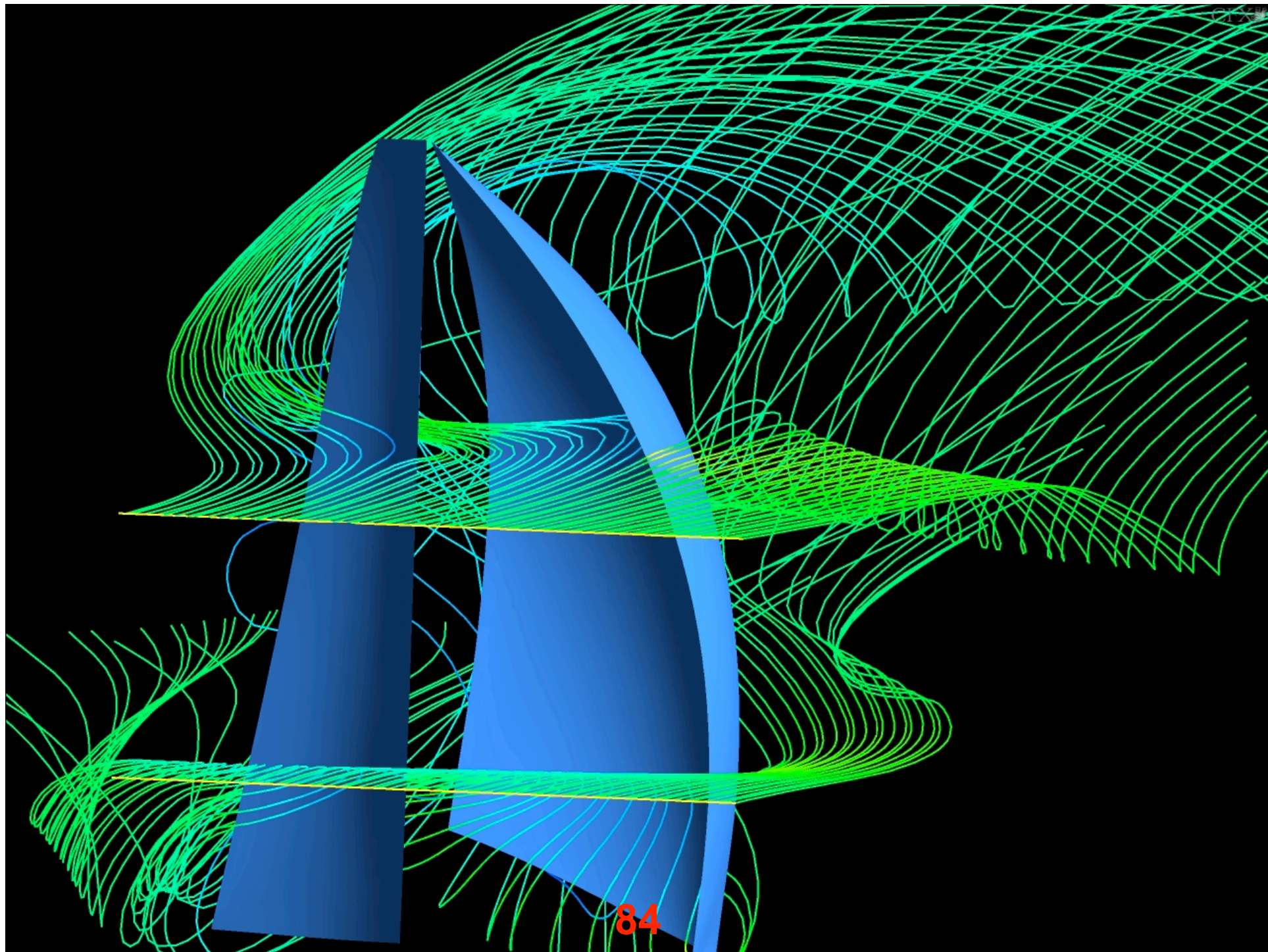
80



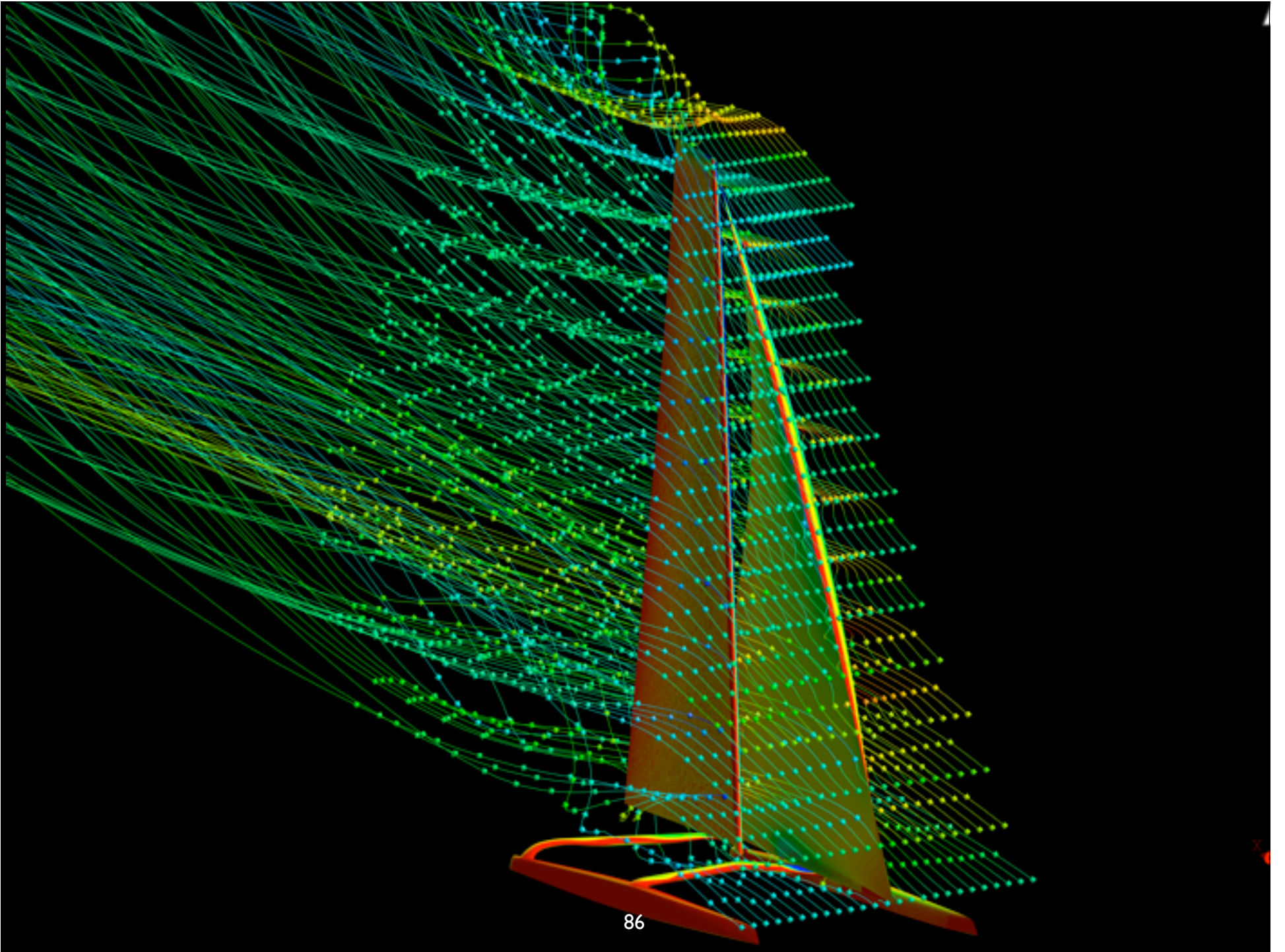


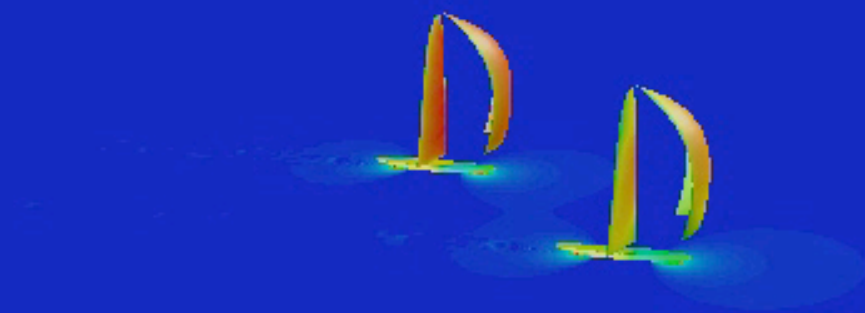
















"Guggenheim Museum Bilbao, Spain" - Photo: David M. Heald © 2002, New York



www.shutterstock.com - 6451804

Muchas Gracias