

Deadline: May 31th

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**Name:**

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## Exercises

1) Write the complete addition table of  $E : y^2 = x^3 + x + 3$  over  $\mathbb{F}_5$  without using the computer.

2) Suppose that in elliptic Diffie-Hellman key exchange with  $E : y^2 = x^3 + 1$  over  $\mathbb{F}_5$  and  $G = (2, 3)$  both parties send  $(0, 1)$ . What is the shared key?

3) We impose in the definition of elliptic curve the condition  $4a^3 + 27b^2 \neq 0$ . Consider for instance  $E : y^2 = x^3 + 2x + 2$  over  $\mathbb{F}_5$  that does not fulfill this condition and show that it does not give a coherent group law.

4) Consider an elliptic curve  $E : y = x^3 + ax + b$  over  $\mathbb{Q}$  at let  $n_2$  the number of points of order exactly 2. Prove that  $n_2 \neq 2$ .

5) An elliptic curve  $E$  over a finite field  $K$  contains 1089 points (including the point at infinity),  $E(K) = \{P_0 = O, P_1, P_2, \dots, P_{1088}\}$ . Suppose that  $P_n = nP_1$  for  $1 \leq n < 1088$ . How many elements are there of each order?

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